$\boxed{5.5}$ Plugging 5.5.19 into 5.5.17, we get

$$\langle \vec{r}_1, \dots, \vec{r}_N | e^{-\beta \hat{H}} | \vec{r}_1, \dots, \vec{r}_N \rangle = \frac{1}{N! \lambda^{3N}} \int d^{3N} r \sum_P (-1)^P \prod_{i=1}^{N} \left(f(P\vec{r}_i - \vec{r}_i) \right)$$

w/ $f(\vec{r}) = e^{-\pi r^2/\lambda^2}$, and $\lambda = \hbar \left(\frac{2\pi\beta}{m} \right)^{1/2}$ by 5.5.18 and 5.5.16 respecivly

and the summation term becomes $1 \pm \sum_{i<j} f_{ij} f_{ji}$ using 5.5.19 and keeping only the first two terms.

$$= \frac{1}{N! \lambda^{3N}} \int d^{3N} r \left(1 \pm \sum_{i<j} f_{ij} f_{ji} \right)$$

Now, $f_{ij} = f(\vec{r}_i - \vec{r}_j) = e^{-\pi(\vec{r}_i - \vec{r}_j)^2/\lambda^2} = e^{-\pi(\vec{r}_j - \vec{r}_i)^2/\lambda^2} = f(\vec{r}_j - \vec{r}_i) = f_{ji}$

$\Rightarrow f_{ij} f_{ji} = f_{ij}^2$

$$\therefore Q_N(V,T) \approx \frac{1}{N! \lambda^{3N}} \int d^{3N} r \left[1 \pm \sum_{i<j} (f_{ij})^2 \right] = \frac{1}{N! \lambda^{3N}} \int d^{3N} r \, e^{\ln\left(1 \pm \sum_{i<j} e^{-2\pi(\vec{r}_i - \vec{r}_j)^2/\lambda^2}\right)}$$

$$\approx \frac{1}{N! \lambda^{3N}} \int d^{3N} r \, e^{\sum_{i<j} \ln(1 \pm e^{-2\pi r^2/\lambda^2})} = \frac{1}{N! \lambda^{3N}} \int d^{3N} r \, e^{-\beta(kT) \sum_{i<j} \ln(1 \pm e^{-2\pi r^2/\lambda^2})}$$

and since $V_s(r_{ij}) = -kT \ln(1 \pm e^{-2\pi r^2/\lambda^2})$ by 5.5.28.

$$\Rightarrow Q_N(V,T) = \frac{1}{N! \lambda^{3N}} \int d^{3N} r \, e^{-\beta \sum_{i<j} V_s(r_{ij})}$$

Then, let $Z_N(V,T) = \int e^{-\beta \sum_{i<j} V_s(r_{ij})} d^{3N} \Rightarrow \boxed{Q_N(V,T) = \frac{1}{N! \lambda^{3N}} Z_N(V,T)}$

$$Q_N = \frac{1}{N! \lambda^{3N}} \int d^{3N} r \left(1 \pm \sum_{i<j} e^{-2\pi(\vec{r}_i - \vec{r}_j)^2/\lambda^2} \right)$$

Carry out integration on first term, and let $(\vec{r}_i - \vec{r}_j)$ be the same for all pairs, which gives us $\frac{N(N-1)}{2}$ unique pairs.

$$Q_N = \frac{1}{N! \lambda^{3N}} V^N \left[1 \pm \frac{N(N-1)}{2} \int_0^\infty e^{-2\pi r_{ij}^2/\lambda^2} 4\pi r^2 dr \right]$$

This integral is evaluated in 5.5.25-26. $\rightarrow \frac{\lambda^3}{2^{3/2} V}$

$$\boxed{Q_N(V,T) = \frac{V^N}{N! \lambda^{3N}} \left[1 \pm \frac{N(N-1)}{2^{5/2}} \left(\frac{\lambda^3}{V} \right) \right]}$$

$\frac{10}{10}$

Let's calculate the first-order correction to A (the Helmoltz Free Energy).

$$A = -kT \ln Q_N = -kT \ln \left[\frac{V^N}{N! \lambda^{3N}} \left(1 \pm \frac{N(N-1)}{2^{5/2}} \left(\frac{\lambda^3}{V} \right) \right) \right]$$ and by 5.5.20, $\frac{V^N}{N! \lambda^{3N}} = \left[\frac{V(2\pi m k T)^{3/2}}{h^3} \right]^N \frac{1}{N!}$

$$A = -kT \left[\ln \left[\frac{V(2\pi m k T)^{3/2}}{h^3} \right]^N - \ln(N!) + \ln \left(1 \pm \frac{N(N-1)\lambda^3}{V 2^{5/2}} \right) \right]$$ use log rules + stirling aprox...

$$\Rightarrow A = NkT \left[\ln \left(\frac{1}{V} \left(\frac{h^2}{2\pi m k T} \right)^{3/2} \right) - \ln N \right] - \boxed{kT \ln \left(1 \pm \frac{N(N-1)}{2^{5/2}} \frac{\lambda^3}{V} \right)}$$ ← first order correction