

15.1

$$\textcircled{a} \text{ Begin w/ 15.1.11: } \Delta S = \frac{C_v}{T} \Delta T + \left. \frac{\partial P}{\partial T} \right|_V \Delta V \Rightarrow \overline{(\Delta T \Delta S)} = \frac{C_v}{T} (\Delta T)^2 + \left. \frac{\partial P}{\partial T} \right|_V \overline{\Delta V \Delta T}$$

$$\text{By 15.1.14a, } \overline{(\Delta T)^2} = \frac{kT^2}{C_v}, \text{ by 15.1.14b, } \overline{\Delta T \Delta V} = 0 \Rightarrow \overline{(\Delta T \Delta S)} = \frac{C_v}{T} \frac{kT^2}{C_v} + 0$$

$$\Rightarrow \boxed{\overline{(\Delta T \Delta S)} = kT}$$

$$\textcircled{b} \text{ Begin w/ 15.1.12: } \Delta P = \left. \frac{\partial P}{\partial T} \right|_V \Delta T - \frac{1}{\chi_T V} \Delta V \Rightarrow \overline{(\Delta P \Delta V)} = \left. \frac{\partial P}{\partial T} \right|_V \overline{\Delta T \Delta V} - \frac{1}{\chi_T V} \overline{(\Delta V)^2}$$

$$\text{By 15.1.14a, } \overline{(\Delta V)^2} = kT \chi_T V, \text{ By 15.1.14b, } \overline{(\Delta T \Delta V)} = 0 \Rightarrow \overline{(\Delta P \Delta V)} = 0 - \frac{kT \chi_T V}{\chi_T V}$$

$$\Rightarrow \boxed{\overline{(\Delta P \Delta V)} = -kT}$$

$$\textcircled{c} \text{ Begin w/ 15.1.11: } \Delta S = \frac{C_v}{T} \Delta T + \left. \frac{\partial P}{\partial T} \right|_V \Delta V \Rightarrow \overline{\Delta S \Delta V} = \frac{C_v}{T} \overline{\Delta T \Delta V} + \left. \frac{\partial P}{\partial T} \right|_V \overline{(\Delta V)^2}$$

$$\text{By 15.1.14a, } \overline{(\Delta V)^2} = kT \chi_T V, \text{ By 15.1.14b, } \overline{(\Delta T \Delta V)} = 0 \Rightarrow \overline{\Delta S \Delta V} = \left. \frac{\partial P}{\partial T} \right|_V kT \chi_T V.$$

$$\text{Now } \chi_T \text{ is defined implicitly through 15.1.12: } \left. \frac{\partial P}{\partial V} \right|_T = -\frac{1}{\chi_T V} \Rightarrow \chi_T = +\frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T$$

$$\Rightarrow \overline{\Delta S \Delta V} = \left. \frac{\partial P}{\partial T} \right|_V kT \left(+\frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T \right) V = \left. \frac{\partial P}{\partial T} \right|_V \left. \frac{\partial V}{\partial P} \right|_T kT = \boxed{\left. \frac{\partial V}{\partial T} \right|_P kT}$$

$$\textcircled{d} \text{ Begin w/ 15.1.12: } \Delta P = \left. \frac{\partial P}{\partial T} \right|_V \Delta T - \frac{1}{\chi_T V} \Delta V \Rightarrow \overline{\Delta P \Delta T} = \left. \frac{\partial P}{\partial T} \right|_V \overline{(\Delta T)^2} - \frac{1}{\chi_T V} \overline{\Delta V \Delta T}$$

$$\text{By 15.1.14a, } \overline{(\Delta T)^2} = \frac{kT^2}{C_v}, \text{ By 15.1.14b, } \overline{\Delta V \Delta T} = 0 \Rightarrow \overline{\Delta P \Delta T} = \left. \frac{\partial P}{\partial T} \right|_V \frac{kT^2}{C_v} - 0$$

$$\Rightarrow \boxed{\overline{\Delta P \Delta T} = \frac{kT^2}{C_v} \left. \frac{\partial P}{\partial T} \right|_V}$$