$\boxed{12.20}$ $\Psi_h(t,m) = -hm + g(t) + r(t)m^2 + s(t)m^4 + u(t)m^6$

ⓐ First minimize Ψ wrt m: $\frac{d\Psi}{dm} = 0 = -\underset{\approx 0}{h} + 2rm + 4sm^3 + 6um^5$

(The ordering field h can be assumed to be negligable) Solve for m:

$$m = \pm \frac{\sqrt{\frac{s}{u} \pm \frac{\sqrt{s^2-3ru}}{u}}}{\sqrt{3}}, \text{ or } m = 0$$

Now, if $r>0$, and $s > -\sqrt{3ur}$, and we recall $u>0$, then $-3ru<0$.
Thus, s^2-3ru will be negative, yielding an imaginary solution $=$ not possible.
$\Rightarrow m_0 = 0$

ⓑ Let's start by simplifying m: $m = \pm\sqrt{\frac{-s \pm \sqrt{s^2-3ru}}{3u}}$ ⊛

Now $r>0$, and $-(4ur)^{1/2} < s \le -(3ur)^{1/2}$. $m_0 = 0$ still works obviously, but now $s^2-3ru \geq 0$, so we get from ⊛

$m_1 = \pm\frac{\sqrt{-s \pm \sqrt{s^2-3ru}}}{\sqrt{3u}}$. However, plugging in $s = -(4ur)^{1/2}$, we get

$m_1 = \pm\left(\frac{r}{u}\right)^{1/4}$ or $m_1 = \pm\sqrt{3}\left(\frac{r}{u}\right)^{1/4}$ These can never be zero (since $r>0$), so $m_0 = 0$ is still the lowest, equilibrium value. Same for $s = -(3ur)^{1/2} \Rightarrow m_1 = \pm\left(\frac{3r}{4}\right)^{1/4}$, and everywhere in between.

plug in to Ψ to ascertain minima info

ⓒ we saw above in (b), that for $s = -(4ur)^{1/2}$, $m_0 = 0$ or $m_1 = \pm\left(\frac{r}{u}\right)^{1/4}$ (this is smaller than the other value: $\pm\sqrt{3}\left(\frac{r}{u}\right)^{1/4}$). The negative value could give this spontaneous magnization.

ⓓ Working w/ m_1^2, let's plug in $s = -(4ur)^{1/2}\alpha$, where $\alpha > 1$ (Thus $s < -(4ur)^{1/2}$)

$$\Rightarrow m_1^2 = \frac{2\sqrt{ru}\,\alpha \pm \sqrt{ru(4\alpha^2-3)}}{3u}$$

Thus we have 2 possible states availibe from the plus or minus sign.

(Note that $2\sqrt{ru}\,\alpha > 0$, $\sqrt{ru(4\alpha^2-3)} > 0$, and $3u > 0$.

ⓔ $r=0$, $s<0 \Rightarrow m_1 = \pm\frac{\sqrt{\frac{s}{u} \pm \frac{|s|}{u}}}{\sqrt{3}} = \begin{cases} \pm\sqrt{\frac{2|s|}{3u}} \\ 0 \end{cases} \Rightarrow m_0 = \pm\sqrt{\frac{2|s|}{3u}}$, or 0

ⓕ Clearly, if $s>0$, and $r\to 0$, then $m \to \sqrt{\frac{-s\pm\sqrt{s^2}}{3u}} \to 0$ (from ⊛)
But for $r<0$ we have a situation where $m \to \sqrt{\frac{-s\pm\sqrt{s^2+\delta}}{3u}}$ $(\delta>0)$, so, as shown before $m_0 = \pm m_1$.

ⓖ If $r=0$, and $s>0$, then $m = \sqrt{\frac{-s\pm\sqrt{s^2}}{3u}} = 0$. (from ⊛)