10.5 By H.1c, $\left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial z}\right|_x \left.\frac{\partial z}{\partial x}\right|_y = -1 \Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{-1}{\left.\frac{\partial P}{\partial H}\right|_T \left.\frac{\partial H}{\partial T}\right|_P}$

By H.1a, $\left.\frac{\partial x}{\partial y}\right|_z = 1 / \left.\frac{\partial y}{\partial x}\right|_z \Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{-\left.\frac{\partial H}{\partial P}\right|_T}{\left.\frac{\partial H}{\partial T}\right|_P}$

$dH = TdS + VdP + \cancel{\mu dN}^{\,0}$ (assume N = constant).

$\Rightarrow \left.\frac{\partial H}{\partial P}\right|_T = T\left.\frac{\partial S}{\partial P}\right|_T + V = -T\left.\frac{\partial V}{\partial T}\right|_P + V$ where the last step was by H.16a

By 1.3.18, $C_P = \left.\frac{\partial H}{\partial T}\right|_P \Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{T\left.\frac{\partial V}{\partial T}\right|_P - V}{C_P}$ ①

By 10.21, $\frac{Pv}{kT} = \sum_{l=1}^{\infty} a_l(T)\left(\frac{\lambda^3}{v}\right)^{l-1} \approx a_1(T) + a_2(T)\left(\frac{\lambda^3}{v}\right) + \cdots = 1 + a_2(T)\left(\frac{\lambda^3}{v}\right)$

$\Rightarrow v = \frac{kT}{P} + \frac{a^2\lambda^3 kT}{Pv}$,

Now, I'm going to make the (sketchy?) substitution $\frac{kT}{Pv} \approx 1$, by 10.21, again.

$\Rightarrow v = \frac{kT}{P} + a^2\lambda^3 \Rightarrow V = \frac{NkT}{P} + Na^2\lambda^3$

Plug V into ①, and get:

$\left.\frac{dT}{dP}\right|_H = -\left[\cancel{\frac{-NkT}{P}} - T\frac{\partial}{\partial T}Na_2\lambda^3 + \cancel{\frac{NkT}{P}} + Na^2\lambda^3\right]\frac{1}{C_P} = \frac{-N}{C_P}\left[-T\frac{\partial(a_2\lambda^3)}{\partial T} + a_2\lambda^3\right]$

$\Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{N}{C_P}\left(T\frac{\partial(a_2\lambda^3)}{\partial T} - a_2\lambda^3\right)$ ✓

Let's compute $\left.\frac{\partial T}{\partial P}\right|_H$ for the given interparticle interaction.

i) $u(r) = \infty$, $0 < r < D$: $\left(a_2 = \frac{2\pi}{\lambda^3}\int_0^\infty (1 - e^{-u(r)/kT})r^2 dr \text{ by } 10.3.1\right)$

$a_2 = \frac{2D^3\pi}{3\lambda^3} \Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{N}{C_P}\left(\frac{2D^3\pi}{3}\left(T\frac{\partial\lambda^3}{\partial T}\frac{1}{\lambda^3} - \frac{\lambda^3}{\lambda^3}\right)\right) = \boxed{\frac{-2ND^3\pi}{3C_P}}$

ii) $u(r) = -u_0$, $D < r < r_1$ $\quad a_2 = \frac{2\pi}{3\lambda^3}(e^{u/kT} - 1)(D^3 - r_1^3)$

(This is the only T-dependant regiem)

$\Rightarrow \left.\frac{\partial T}{\partial P}\right|_H = \frac{N}{C_P}\left(T\frac{\partial}{\partial T}\left(\frac{2\pi}{3}(e^{u_0/kT} - 1)(D^3 - r_1^3)\right) - \frac{2\pi}{3}(e^{u_0/kT} - 1)(D^3 - r_1^3)\right)$

$\Rightarrow \boxed{\left.\frac{\partial T}{\partial P}\right|_H = \frac{2N\pi}{3C_P kT}(D^3 - r_1^3)(kT - e^{u_0/kT}(kT + u_0))}$

iii) $u(r) = 0$, $r > r_1$ $\Rightarrow a_2 = 0 \Rightarrow \boxed{\left.\frac{\partial T}{\partial P}\right|_H = 0}$ temp limits?

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