$\boxed{10.3}$ Given (10.3.9): $(P+\frac{a}{v^2})(v-b) \simeq kT$

ⓐ Solve for P, we get $P = \frac{kT}{v-b} - \frac{a}{v^2}$ ①

The goal is to get $C_p - C_v$. We will assume N is constant. Then $C_v = \frac{\partial U}{\partial T}\Big|_V$, $C_p = \frac{\partial U}{\partial T}\Big|_P$

Let's get U.

By 10.7.8, $P = -\frac{\partial A}{\partial V}\Big|_T = \frac{kT}{v-b} - \frac{a}{v^2} \Rightarrow -A = \int dV\left[\left(\frac{kT}{v-b}\right) - \frac{a}{v^2}\right]$

$\Rightarrow -A = kTN\ln(V-bN) - \frac{aN^2}{V} + \mathcal{K}(T)$

where I have noted $v = \frac{V}{N}$, and carried out the integration. $\mathcal{K}(T)$ is a constat of integration, which may still have Temperature dependance.

By 3.3.5, $U = T^2\left[\frac{\partial}{\partial T}\left(\frac{-A}{T}\right)\right]_V = -T^2\frac{\partial}{\partial T}\left(\frac{Na}{vT}\right) + T^2\frac{\partial}{\partial T}\left(\frac{\mathcal{K}(T)}{T}\right) = \frac{Na}{v} - \mathcal{K}'(T)$

Thus, we have $C_v = \frac{\partial U}{\partial T}\Big|_V = -\mathcal{K}''(T) + Na\frac{\partial}{\partial T}\left(\frac{1}{v}\right)\Big|_V = -\mathcal{K}''(T) + \frac{Na}{v^2}\frac{\partial v}{\partial T}\Big|_V$

$C_p = \frac{\partial U}{\partial T}\Big|_P = -\mathcal{K}''(T) - Na\frac{\partial}{\partial T}\left(\frac{1}{v}\right)\Big|_P = -\mathcal{K}''(T) - \frac{Na}{v^2}\frac{\partial v}{\partial T}\Big|_P$

$\Rightarrow C_p - C_v = \frac{-Na}{v^2}\frac{\partial v}{\partial T}\Big|_P \overset{\text{By ①}}{=} N\left(\frac{kT}{v-b}\right)\frac{\partial v}{\partial T}\Big|_P - \cancel{NP\frac{\partial v}{\partial T}\Big|_P}^{\,0}$ ②

Now we find $\frac{\partial v}{\partial T}\Big|_P$

Using ①, $\cancel{\frac{\partial P}{\partial T}\Big|_P}^{\,0} = \frac{k}{v-b} + \left(\frac{2a}{v^3} - \frac{kT}{(v-b)^2}\right)\frac{\partial v}{\partial T}\Big|_P \Rightarrow \frac{\partial v}{\partial T}\Big|_P = \frac{k}{v-b}\left(\frac{1}{\frac{kT}{(v-b)^2} - \frac{2a}{v^3}}\right)$

Using ②

$\Rightarrow C_p - C_v = \frac{NkT}{v-b}\,\frac{k}{v-b}\left(\frac{kT}{(v-b)^2} - \frac{2a}{v^3}\right)^{-1} = \boxed{Nk\left(1 - \frac{2a(v-b)^2}{v^3\,kT}\right)^{-1}}$

ⓑ By H.6b, $S = \frac{-\partial A}{\partial T}\Big|_V$, and from part a, $-A = kTN\ln(v-b) - \frac{Na}{v} + \mathcal{K}(T)$

$\Rightarrow S = kN\ln(v-b) + \mathcal{K}'(T)$ ③

We saw in part a that $U = \frac{Na}{v} - T^2\frac{\partial}{\partial T}\left(\frac{\mathcal{K}(T)}{T}\right) = \frac{Na}{v} - T^2\left(\frac{T\mathcal{K}'(T)}{T^2} - \frac{\mathcal{K}(T)}{T^2}\right)$

$\Rightarrow U = \frac{Na}{v} - \mathcal{K}'(T)\cdot T + \mathcal{K}(T)$

Again $C_v = \frac{\partial U}{\partial T}\Big|_V = \cancel{\mathcal{K}'(T)} - \mathcal{K}''(T)T - \cancel{\mathcal{K}'(T)} = -\mathcal{K}''(T)T$

$\Rightarrow \frac{\partial^2 \mathcal{K}(T)}{\partial T^2} = -\frac{C_v}{T} \Rightarrow \frac{\partial \mathcal{K}(T)}{\partial T} = C_v\ln(T) + \text{constant}.$

For adiabatic processes, S = constat. Using ③, and rolling the constats together,

$-kN\ln(v-b) = C_v\ln(T) + \text{const.}$

$\Rightarrow \text{const.} = kN\ln(v-b) + C_v\ln(T) = \ln[(v-b)^{Nk}] + \ln(T^{C_v})$

$\text{Other const.} = (v-b)^{Nk}\,T^{C_v} = [(v-b)\,T^{C_v/Nk}]^{Nk}$

$\boxed{(v-b)\,T^{C_v/Nk} = \text{Constant.}}$

10.3 (c) Expanding into a vaccum requires no work or heating.
$\Rightarrow U = \text{constant}.$

from (a): $U = \frac{Na}{v} - \chi'(T) = \frac{Na}{v} - C_v T + \text{constant}.$

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Then we make 2 copies: $U_2 = \frac{N^2 a}{V_2} - C_v T_2$, $U_1 = \frac{N^2 a}{V_1} - C_v T_1$

$U_2 = U_1 \Rightarrow \frac{N^2 a}{V_2} - C_v T_2 = \frac{N^2 a}{V_1} - C_v T_1 \Rightarrow \boxed{T_2 - T_1 = \frac{N^2 a}{C_v}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)}$