$\boxed{\begin{array}{l}\#1\\ \text{Goldstein 5-6}\end{array}}$ (a) Since the top is symmetrical, we know $I_1 = I_2 \equiv I$, while I_3 is something else that we don't know. Substituting this into equation 5.47, The two non-trivial equations are:

$$I\dot{\omega}_1 = (I - I_3)\,\omega_3\,\omega_2 \qquad \text{and} \qquad I\dot{\omega}_2 = -(I - I_3)\,\omega_3\,\omega_1$$

$\Rightarrow$ $\dot{\omega}_1 = \Omega\,\omega_2$ and $\dot{\omega}_2 = -\Omega\,\omega_1$ where $\Omega = \frac{I - I_3}{I}\omega_3$, which is a constant. (Note $\omega_3 = \text{constant}$, since the top's primary rate of rotation about it's body symmetry axis is fixed.)

Solving these differential equations...

$$\ddot{\omega}_1 = \Omega\,\dot{\omega}_2 = -\Omega^2\,\omega_1 \Rightarrow \omega_1 = A\cos(\Omega t)$$

and plug $\omega_1 = A\cos(\Omega t)$ into $\dot{\omega}_1 = \Omega\,\omega_2$

$$\Rightarrow \cancel{\Omega} A\sin(\Omega t) = \cancel{\Omega}\,\omega_2 \Rightarrow \omega_2 = A\sin(\Omega t)$$

So in the body coordinates, Angular Momentum is:

$$L_1 = I\omega_1 = I A\cos(\Omega t)$$
$$L_2 = I\omega_2 = I A\sin(\Omega t)$$
$$L_3 = I_3\,\omega_3 = \text{constant} \equiv K$$

Thus $\boxed{\vec{L} = IA(\sin(\Omega t)\hat{i} + \cos(\Omega t)\hat{j}) + K\hat{k}}$, indicating that the top's angular momentum vector rotates about the symmetry axis at a frequency Ω. (no ext. torques, so it stays this way).

Now we relate the body coordinates to the Lab-frame Euler angles ϕ, θ, ψ.

By eq. 4.87:

$$\omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi = A\cos(\Omega t)$$
$$\omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi = A\sin(\Omega t)$$
$$\omega_3 = \dot{\phi}\cos\theta + \dot{\psi} = K$$

In the lab frame, $\dot{\psi}$ describes the same thing that Ω does in the body frame. So

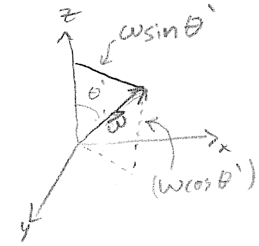
$$\omega_3 = \dot{\phi}\cos\theta + \Omega \Rightarrow \dot{\phi} = \frac{\omega_3 - \Omega}{\cos\theta}$$

Plug in $\Omega = \frac{I - I_3}{I}\omega_3$ $\Rightarrow$ $\dot{\phi} = \dfrac{\omega_3 - \omega_3\left(\frac{I}{I} - \frac{I_3}{I}\right)}{\cos\theta} = \dfrac{\omega_3\left(1 - 1 + \frac{I_3}{I}\right)}{\cos\theta}$

$$\Rightarrow \boxed{\dot{\phi} = \frac{\omega_3 I_3}{I_1\cos\theta}} \quad ✓ \quad \leftarrow \text{b/c we set } I_1 \equiv I.$$

5-6 cont.

(b) Note first, as shown @ right that for a vector $\vec{\omega}$ w/ polar angle θ', $\omega \sin\theta' = \sqrt{\omega_x^2 + \omega_y^2}$



(I'm switching to x, y, z notation per problem 4-15)

given: $\omega_x = \dot\theta\cos\phi + \dot\psi\sin\theta\sin\phi = \Omega\sin\theta\sin\phi$

$\omega_y = \dot\theta\sin\phi - \dot\psi\sin\theta\cos\phi = -\Omega\sin\theta\cos\phi$

$\dot\theta = 0$, $\dot\psi = \Omega$ $\quad \omega_z = \dot\psi\cos\theta + \dot\phi = \Omega\cos\theta + \dfrac{\omega_3 I_3}{I_1\cos\theta}$ = constant (as seen in (a)).

The angle down from $\vec{L}$ to $\vec{\omega}$, we are told is θ', and that means

$$\omega\sin\theta' = \sqrt{\omega_x^2+\omega_y^2} = \sqrt{\Omega^2\sin^2\theta\sin^2\phi + \Omega^2\sin^2\theta\cos^2\phi} = \Omega\sin\theta$$

$$\Rightarrow \sin\theta' = (\Omega\sin\theta)\frac{1}{\omega}$$

Similarly, the angle down from $\vec{\omega}$ and the symmetry axis, θ, is:

$$\omega\sin\theta'' = \sqrt{\omega_x^2+\omega_y^2} \overset{\text{part a}}{=} \sqrt{A^2\cos^2(\Omega t) + A^2\sin^2(\Omega t)} = A$$

$$\Rightarrow \frac{1}{\omega} = \frac{\sin\theta''}{A}$$

And plug into $\sin\theta' = (\Omega\sin\theta)\frac{1}{\omega}$ to get $\sin\theta' = \dfrac{\Omega\sin\theta\sin\theta''}{A}$

$$\sin\theta' = \Omega\sin\theta''\left(\frac{\sin\theta}{A}\right)$$

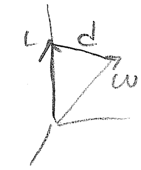
From part (a), $\dot\phi\sin\theta\cos\psi - \underbrace{\dot\theta}_{0}\sin\psi = A\sin(\Omega t)$

$\dot\phi\sin\theta\cos\psi = A\sin(\Omega t) \Rightarrow \dfrac{\sin\theta}{A} = \dfrac{\cancel{\sin(\Omega t)}}{\cancel{\cos(\psi)}}\dfrac{1}{\dot\phi}$

so $\boxed{\sin\theta' = \dfrac{\Omega\sin\theta''}{\dot\phi}}$ ✓

In the section, we are given $\dfrac{I_3 - I_1}{I_1} = 0.00327$

Since $\Omega = \dfrac{I - I_3}{I}\omega_3 = -\left(\dfrac{I_3 - I_1}{I}\right)\omega_3 = -0.00327\,\omega_3$



$$\sin\theta' = \frac{I - I_3}{I}\,\cancel{\omega_3}\,\frac{I\cos\theta}{\cancel{\omega_3}\,I_3}\sin\theta''$$

when we realize how big earth is relative to those angles, we approximate $\sin\theta' \approx \theta'$, $\sin\theta'' \approx \theta''$, $\cos\theta = 1$

Also when R is earth's radius, ℓ = the average seperation distance given in the text as having an amplitude of 10m. (so $\ell \approx 5$m, I guess), and d = the quantity we're finding, $\theta' = \frac{d}{R}$, $\theta'' = \frac{\ell}{R}$

$\dfrac{d}{\cancel{R}} = -0.00327\,\dfrac{\ell}{\cancel{R}}\left(\dfrac{I}{I_3}\right)$ Note $I_3 - I_1 = 0.00327 I_1 \Rightarrow \dfrac{I_3}{I_1} = 1.00327$

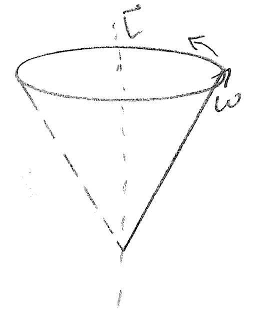
so $|d| = 0.00327(1.00327)(5\text{m}) = 0.0164\text{m} \approx 1.6\text{cm}$ is the maximum.

Ok - so that's close. It'll never be further than $\boxed{1.6\text{cm}.}$

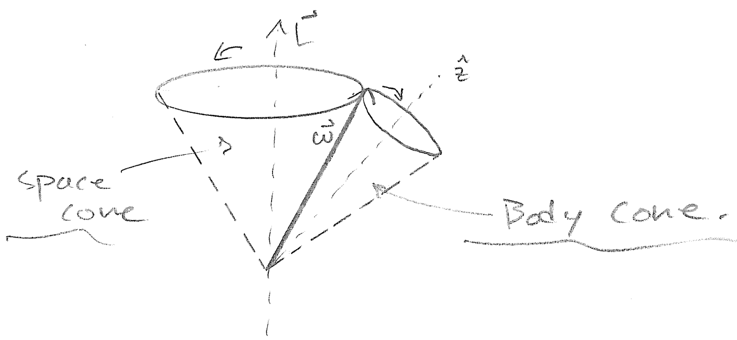
(5-6 cont.)

(c) We know that T and $\vec{L}$ are conserved here (in the absence of exterend torques.) AND, $T = \frac{1}{2}\vec{\omega}\cdot\vec{L}$. Thus, the angle of the ω vector can't be changing in time either. As it whips around the symmetry axis, we say that its magnitude traces out a cone:

This is "space cone."



Meanwhile, the ridgid body's frame has $\vec{\omega}$ (The same $\vec{\omega}$!) precessing around the Body's $\hat{z}$ axis. In this frame too, we observe the $\angle$ that $\vec{\omega}$ traces out to be constant. This is another cone, but $\vec{\omega}$ is always @ the intersection:



Poinsot?

ok

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