1-8 Given: L is the Lagrangian for a system w/ n dof's. Thus,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

Prove: $L' = L + \frac{dF(q_1, \ldots q_n, t)}{dt}$. Show L' Satisfies Lagrange's Eq where F is an abitrary, diffetiable fn. of its arguments.

$$\frac{d}{dt}\left(\frac{\partial L'}{\partial \dot q_j}\right) - \frac{\partial L'}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j} + \frac{\partial}{\partial \dot q_j}\left(\frac{dF}{dt}\right)\right) - \left(\frac{\partial L}{\partial q_j} + \frac{\partial}{\partial q_j}\left(\frac{dF}{dt}\right)\right)$$

By definition of L'.

$$= \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j}\right) - \frac{\partial L}{\partial q_j}}_{0} + \frac{d}{dt}\left(\frac{\partial}{\partial \dot q_j}\left(\frac{dF}{dt}\right)\right) - \frac{\partial}{\partial q_j}\left(\frac{dF}{dt}\right)$$

By the given assuption that L satisfies Lagrange's Equations.

Now, expand the dF/dt's using chain rule:

$$= \frac{d}{dt}\left(\frac{\partial}{\partial \dot q_j}\left(\frac{\partial F}{\partial q_j}\frac{\partial q_j}{\partial t} + \frac{\partial F}{\partial t}\right)\right) \quad \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial q_j}\cdot\frac{\partial q_j}{\partial t} + \frac{\partial F}{\partial t}\frac{dt}{dt}\right)$$

$$= \frac{d}{dt}\left(\frac{\partial}{\partial \dot q_j}\left(\frac{\partial F}{\partial q_j}\dot q_j + \cdots\right)\right) - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial q_j}\cdot\frac{\partial q_j}{\partial t}\right) - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial t}\frac{dt}{dt}\right)$$

carrying out the derivative of $\dot q_j$ w/ respect to $\dot q_j$

$$= \frac{d}{dt}\left(\frac{\partial F}{\partial q_j}\right) - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial q_k}\cdot\frac{\partial q_k}{\partial t}\right) - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial t}\frac{dt}{dt}\right)$$

$$= \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial q_j}\right)\frac{dt}{dt} + \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial q_j}\right)\frac{\partial q_j}{\partial t} - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial q_j}\right)\frac{\partial q_j}{\partial t} - \frac{\partial}{\partial q_j}\left(\frac{\partial F}{\partial t}\cdot\frac{dt}{dt}\right)$$

comment on uniqueness of L?

The middle two terms obviously cancel, and assuming F is "well-behaved" we use Cairaut's Theorm to reverse the order of partials, and the 1st and last term cancel.

$= 0$. Thus L' Satisfies Lagrange's equation.

8/10