#1: Jackson 5.6

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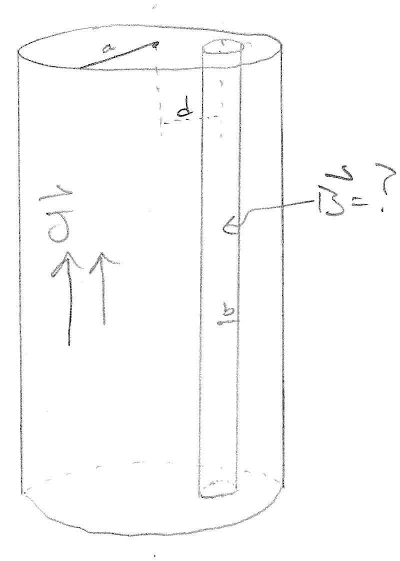


Ampère's Law says that $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

where $I_{enc} = \int \vec{J}\cdot da$ → the current enclosed by some loop.

→ Draw an Amperian loop radius ρ parallel to the "caps" of the cylinder. Ignore the cut-out region for now, and note that in this case $I_{enc,a} = \int \vec{J}\cdot d\vec{a} = J\cdot \pi\rho^2$.

Since $\vec{B}$ and $d\vec{l}$ are parallel for such an amperian loop, we have $B_a\, 2\pi\rho = \mu_0 J\pi\rho^2 \Rightarrow \vec{B}_a = \frac{\mu_0}{2}(\vec{J}\times\vec{\rho})$

→ similarly, we can draw an amperian loop centered around the center of the cavity, and imagine $-\vec{J}$ running through it. Call that radial coordinate $\vec{r}$.

Then $\vec{B}_b = -\frac{\mu_0}{2}(\vec{J}\times\vec{r})$

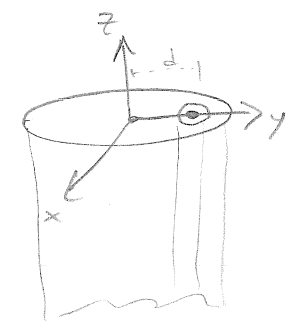
→ But $\vec{r}+\vec{d} = \vec{\rho}$, $\Rightarrow$ $\vec{r} = \vec{\rho}-\vec{d}$ $\Rightarrow$ $\vec{B}_b = \frac{-\mu_0}{2}\left(\vec{J}\times(\vec{\rho}-\vec{d})\right)$

We know that the total B-field is the superposition of $\vec{B}_a$ and $\vec{B}_b$.

$$\vec{B} = \vec{B}_a + \vec{B}_b = \frac{+\mu_0}{2}(\vec{J}\times\vec{\rho}) - \frac{\mu_0}{2}\left(\vec{J}\times(\vec{\rho}-\vec{d})\right)$$

$$\vec{B} = \frac{\mu_0}{2}(\vec{J}\times\vec{d})$$

Assuming our cylinder is lined up like this →
$(\hat{J}\times\hat{d}) = -\hat{x}$, so



$$\boxed{\vec{B} = \frac{-\mu_0}{2} d J \hat{x}}$$ ✓

(10)