$\boxed{4}$ ⓐ The trial function is $\psi(x,y) = A(1-x)\cdot y(1-y)$

Jackson 1.21

As required by 1.63, we need $\nabla\psi$

$$\nabla\psi = A[(1-x)\cdot y(1-y) - x\cdot y(1-y),\ (1-y)\cdot x(1-x) - yx(1-x),\ 0]$$

and $\nabla\psi\cdot\nabla\psi = A^2(y^2(1-y)^2(1-2x)^2 + x^2(1-x)^2(1-2y)^2)$

from 1.63

$$I[\psi] = \frac{1}{2}\int_V \nabla\psi\cdot\nabla\psi\, d^3x - \int g\,\psi\, d^3x$$

we set $g=1$ b/c V does not vary over the specified square.

$$I[\psi] = \frac{1}{2}\int_V d\vec{x}\, A^2(y^2(1-y)^2(1-2x)^2 + x^2(1-x)^2(1-2y)^2) - \int_V d\vec{x}\, A(1-x)\,y(1-y$$

$$I[\psi] = \frac{1}{2}\left(\frac{A^2}{45}\right) - \frac{A}{36}$$

To find the min value of A do $\frac{\partial I}{\partial A} = 0 \implies -\frac{1}{36} + \frac{A}{45} = 0$

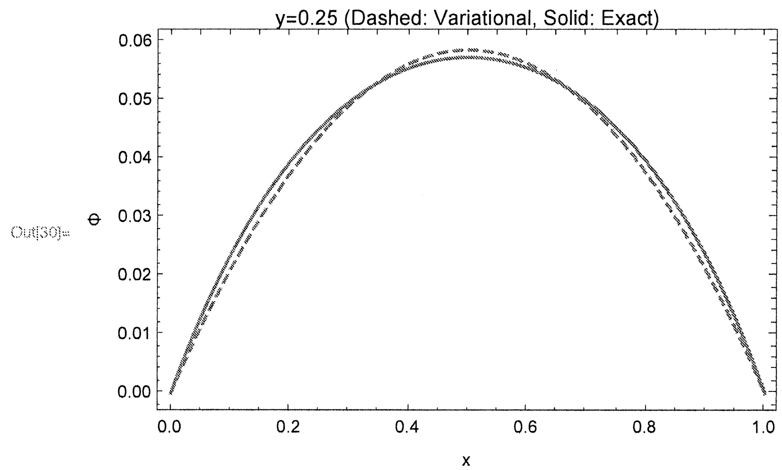
$$\implies \boxed{A = \frac{5}{4}}\ ✓$$

ⓑ See attached. I plotted the two functions for each y-value, using m=100 as my highest index, and the predicted potentials are very similar. ✓

In[15]:=
```
var[x_, y_] = 5 / 4 * x (1 - x) * y * (1 - y);
exact[x_, y_] = 4/π^3 Sum[Sin[(2 m + 1) π * x]/(2 m + 1)^3 (1 - Cosh[(2 m + 1) π (y - 1/2)]/Cosh[(2 m + 1) π/2]), {m, 0, 100}];
```

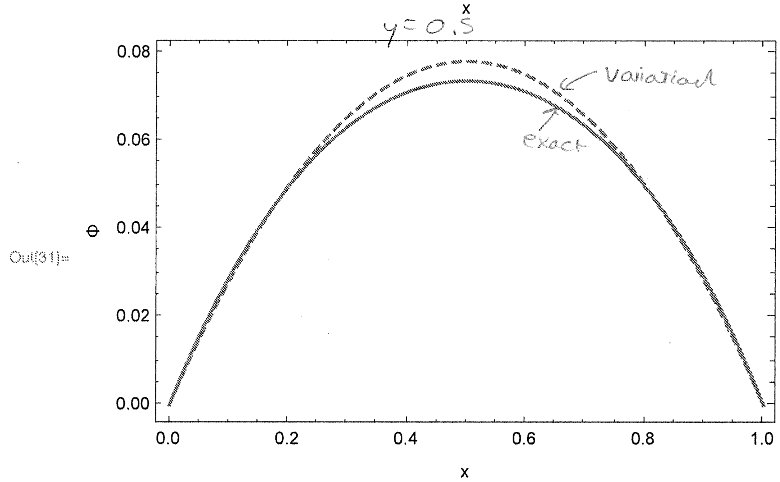
In[30]:=
```
Show[{Plot[var[x, 0.25], {x, 0, 1}, PlotStyle → Dashed],
   Plot[exact[x, 0.25], {x, 0, 1}]}, Frame → True,
  PlotLabel → "y=0.25  (Dashed: Variational, Solid: Exact)", FrameLabel → {"x", "Φ"}]
Show[{Plot[var[x, 0.5], {x, 0, 1}, PlotStyle → Dashed],
   Plot[exact[x, 0.5], {x, 0, 1}]}, Frame → True,
  FrameStyle → Directive[Thick, Black] PlotLabel →
    "y=0.5  (Dashed: Variational, Solid: Exact)", FrameLabel → {"x", "Φ"}]
```

Out[30]=



✓

Out[31]=



✓

10/10