Jackson 1.12

From equation 1.35:

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) d^3x = \int_S \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}\right) da$$

Now, set $\psi \to \phi'$

we know $\nabla^2 \phi = -\rho/\varepsilon_0$, and $\nabla^2 \phi' = -\rho'/\varepsilon_0$

as well as $\sigma = \varepsilon_0 \frac{\partial \phi}{\partial n}$ and $\sigma' = \varepsilon_0 \frac{\partial \phi'}{\partial n}$.

Plug these in and we get:

$$\int_V (\phi \nabla^2 \phi' - \phi' \nabla^2 \phi) d^3x = \int_S \left(\phi \frac{\partial \phi'}{\partial n} - \phi' \frac{\partial \phi}{\partial n}\right) da$$

$$\int_V (\phi \rho - \phi \rho') d^3x = \int_S (\phi \sigma' - \sigma \phi') da$$

Then break up integrals:

$$\int_V \rho \phi' d^3x + \int_S \sigma \phi' da = \int_V \rho' \phi d^3x + \int_S \sigma' \phi da \quad \square$$

10/10