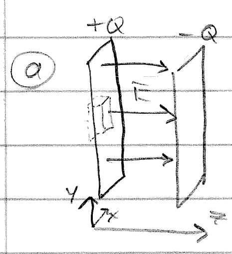
We are asked to compute Capacitance: $C = q/V$ in

Jackson 1.6 the following situations.

(a)



Draw the shown Gaussian pillbox of area A and tiny thickness. This eliminates not normal components of $E \cdot A$. Thus $\int E \cdot d\vec{A} = E \cdot A$

$$\underbrace{E_{Q+}(2A)}_{\text{2 sides}} = \frac{1}{\varepsilon_0} Q_{enc.} \longrightarrow 2E_{Q+} A = \frac{1}{\varepsilon_0} Q \quad \text{when } A \text{ goes big}$$

$$E_{Q+} = \frac{Q}{2\varepsilon_0 A}$$

Similarly $E_{Q-} = \frac{-Q}{2\varepsilon_0 A} \Rightarrow \vec{E} = E_{Q+} + E_{Q-} = \frac{Q}{\varepsilon_0 A}\hat{z}$

Now find V:

$$V(z) = -\int_0^d \frac{Q}{\varepsilon_0 A}\, dz = \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{q}{V} = Q\left(\frac{\varepsilon_0 A}{Qd}\right) \Rightarrow \boxed{C = \frac{\varepsilon_0 A}{d}} \checkmark$$

(b) This time our Gaussian surface is a sphere with radius $a < r < b$.

No E-field from outer shell.

spherical symmetry.

$$E \cdot A = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$V(r) = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$C = \frac{Q}{V} = \frac{-4\pi\varepsilon_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} = \boxed{\frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}} \checkmark$$

Solution Continues on the Next Page.

(c) This time use a gaussian cylinder of radius r, $a<r<b$, which has radius $2\pi r L$.
Again there is cylindrical symmetry, and all E-field is from inner cylinder (charge Q).

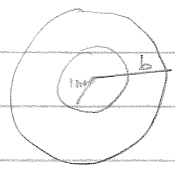
$$E \cdot A = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{\varepsilon_0} \cdot \frac{1}{2\pi r L}$$

Thus

$$V(r) = -\int_a^b \frac{Q}{2\pi r L \varepsilon_0}\, dr = \frac{-Q}{2\pi r L} \ln(r)\Big|_a^b = \frac{-Q}{2\pi L \varepsilon_0} \ln(a/b)$$

$$C = \frac{Q}{V} = \boxed{\frac{2\pi \varepsilon_0 L}{\ln(b/a)}}$$ ✓

(d)



from (c), $C = \dfrac{2\pi\varepsilon_0 L}{\ln(b/a)}$

$$\Rightarrow \ln(b) = \frac{2\pi\varepsilon_0 L}{C} + \ln(a)$$

so $b = a e^{2\pi\varepsilon_0 L/C}$

we plug in $a = 1\text{mm}$, $\frac{\varepsilon_0}{m} = 8.85 \cdot 10^{-12}\, C^2/N \Rightarrow \frac{\varepsilon_0}{L} = 0.885 \cdot 10^{-11}\, F/m$

10/10

Then for $C = 3 \cdot 10^{-11}\, F/m$, $\boxed{b = 6.4\text{mm}}$
for $C = 3 \cdot 10^{-12}\, F/m$, $\boxed{b = 1.2 \cdot 10^{8}\text{mm}}$ ✓

Wow! That's a big difference.