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Jackson 1.3

(a) Since there is a discontinuity only in the $\hat{r}$ component of ρ (ϕ, θ are constant @ any given r), ρ is of the form

$\rho(r, \theta, \phi) = A\,\delta(r-R)$

integrate to find A, given total charge Q.

$$Q = \int_V A\,\delta(r-R)\,dV = A\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\delta(r-R)\,r^2\,dr\,\sin\theta\,d\theta\,d\phi$$

$$= 4\pi R^2 A \Rightarrow A = Q/4\pi R^2$$

$$\boxed{\rho(r,\theta,\phi) = \frac{Q}{4\pi R^2}\,\delta(r-R)}$$ ✓

(b) Now z, ϕ are constant, but the discontinuity is @ $r=b$.

$\rho(r,\phi,z) = A\,\delta(r-b)$

we only do a surface integral (over ϕ, r) b/c we have a linear charge density which already accounts for z.

$$\lambda = \int_{Area}\delta(r-b)\,dA = A\int_0^{\infty}\int_0^{2\pi}\delta(r-b)\,r\,d\theta\,dr = A2\pi b \Rightarrow A = \frac{\lambda}{2\pi b}$$

$$\boxed{\rho(r,\phi,z) = \frac{\lambda}{2\pi b}\,\delta(r-b)}$$ ✓

(c) Here ρ is of the form $\rho(r,\phi,z) = A\,\delta(z)\,\Theta(R-r)$, where Θ is the step function with a value of 1 for $0 \le r \le R$, and 0 for $r > R$. This is discontinuity in the r-direction. $\delta(z)$ accounts for the discontinuity of the disk vertically.

$$Q = \int_V A\,\delta(z)\,\Theta(R-r)\,dV = A\int_{-\infty}^{\infty}\delta(z)\int_0^{2\pi}\int_0^{\infty} r\,\Theta(R-r)\,dr\,d\phi\,dz$$

$$= A\cdot 2\pi\int_0^{\infty} r\,\Theta(R-r)\,dr = A\pi R^2 \Rightarrow A = \frac{Q}{\pi R^2}$$

$$\boxed{\rho(r,\phi,z) = \frac{Q}{\pi R^2}\,\delta(z)\,\Theta(R-r)}$$ ✓

(d) In the spherical coordinate system, we notice $\rho \propto \frac{\pi r^2}{4\pi r^3} \propto \frac{1}{r}$, b/c as we grow out a sphere of radius r, it encompases less and less charge per volume. Clearly ρ falls off like $1/r$.

thus, we stick this in our guess for ρ.

$$\rho(r,\theta,\phi) = \frac{1}{r} A\, \Theta(R-r)\, \delta(\theta - \tfrac{\pi}{2})$$

otherwise, we just replace $\delta(z)$ w/ $\delta(\theta - \frac{\pi}{2})$, which amounts to the same thing in spherical coordinates.

$$Q = A \int_0^{2\pi} \left(\int_0^{\pi} \left[\int_0^{\infty} \frac{1}{r} \Theta(R-r)\, r^2\, dr \right] \delta(\theta - \tfrac{\pi}{2}) \sin(\theta)\, d\theta \right) d\phi$$

$$Q = 2\pi A \int_0^R r\, dr \Rightarrow Q = \pi A R^2 \Rightarrow A = \frac{Q}{\pi R^2}$$

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$$\boxed{\rho(r,\theta,\phi) = \frac{Q}{\pi R^2} \frac{\Theta(R-r)}{r} \delta(\theta - \tfrac{\pi}{2})}$$

✓ (different from what I have, but I think this works)