+3/3

45.1 Begin w/ the definition of power radiated by an object w/ both an electric & a magnetic dipole moment:

32.38: $P_{rad} = \frac{2}{3c^3}\left((\ddot{\vec{d}})^2 + (\ddot{\vec{\mu}})^2\right)$

Now we follow the procedure in Schwinger section 45.3

→ The electric dipole mom of a dielectric sphere is given by 45.32: $\vec{d} = \frac{\varepsilon-1}{\varepsilon+2} a^3 \vec{E}$

for a perfect conductor, $\varepsilon \to \infty$, so $\lim_{\varepsilon\to\infty}\left(\frac{\varepsilon-1}{\varepsilon+2} a^3 \vec{E}\right) = a^3\vec{E} \Rightarrow \vec{d} = a^3\vec{E}$.

→ The magnetic dipole moment of a conducting sphere was found in problem 28.2 to be $\vec{\mu} = -\frac{1}{2}a^3\vec{B}$

→ Assume $\vec{E}$ and $\vec{B}$ vary in time w/ frequency ω. Then $\vec{E} = \vec{E}_0 e^{i\omega t}$, $\vec{B} = \vec{B}_0 e^{i\omega t}$.

Then, $\ddot{\vec{d}} = (i\omega)(i\omega)a^3\vec{E}_0 e^{i\omega t} = -\omega^2 a^3 \vec{E} \Rightarrow (\ddot{\vec{d}})^2 = \omega^4 a^6 \vec{E}^2$ ✓

and similarly, $(\ddot{\vec{\mu}})^2 = \frac{\omega^4}{4} a^6 \vec{B}^2$

$\Rightarrow P_{rad} = \frac{2\omega^4 a^6}{3c^3}\left(\frac{\vec{B}^2}{4} + \vec{E}^2\right) \Rightarrow \overline{P_{rad}} = \frac{2\omega^4 a^6}{3c^3}\left(\frac{1}{4}\overline{\vec{B}^2} + \overline{\vec{E}^2}\right)$

where the bars denote time-averaging.

We note now that $\overline{\vec{B}^2} = \overline{\vec{E}^2}$, so we have $\overline{P_{rad}} = \frac{2\omega^4 a^6}{3c^3}\left(\frac{5}{4}\right)\overline{\vec{E}^2}$ ✓

→ By 45.24 $\overline{\vec{E}^2} = \frac{4\pi}{c}|\vec{S}| \Rightarrow \overline{P_{rad}} = \frac{8\pi\omega^4 a^6}{3c^4}\left(\frac{5}{4}\right)|\vec{S}|$

Finally, by 45.4, the scattering cross-section is $\sigma = \frac{P_{scat}}{|\vec{S}|}$

$\Rightarrow \boxed{\sigma = \frac{8\pi}{3c^4}\omega^4 a^6\left(\frac{5}{4}\right)}$ ✓ (Note this doesn't coincide with $\sigma = \frac{8\pi}{3c^4}\omega^4 a^6$, which is the $\varepsilon\to\infty$ limit of 45.35.)