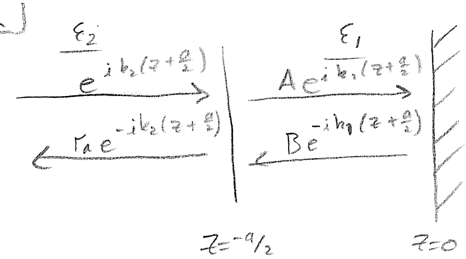
2012 EMII - 3 | Identical to Schwinger E&M problem 41.2

For $z < -\frac{a}{2}$, $\quad g(z) = e^{ik_2(z+\frac{a}{2})} + r_a e^{-ik_2(z+a/2)}$

$0 > z > \frac{a}{2}$, $\quad g(z) = Ae^{ik_1(z+\frac{a}{2})} + Be^{-ik_1(z+a/2)}$



Match Boundary Conditions

1) $g(z=0) = 0 \implies Ae^{ik_1 a/2} + Be^{-ik_1 a/2} = 0 \implies B = -Ae^{ik_1 a}$

2) g is continuous @ $z = -a/2$: $\quad 1 + r_a = A + B$

3) $\frac{dg}{dz}$ is continuous @ $z = -\frac{a}{2}$ $\quad k_2(1 - r_a) = k_1(A - B)$

Now solve for r_a

$$1 + r_a = A(1 - e^{ik_1 a}) \quad , \quad k_2(1 - r_a) = Ak_1(1 + e^{ik_1 a})$$

$$A = \frac{1 + r_a}{1 - e^{ik_1 a}} \implies \frac{k_2}{k_1}\frac{(1 - r_a)}{1 + e^{ik_1 a}} = \frac{1 + r_a}{1 - e^{ik_1 a}}$$

$$\implies \frac{k_2}{k_1}\frac{1 - e^{ik_1 a}}{1 + e^{ik_1 a}} - \frac{k_2}{k_1}\frac{1 - e^{ik_1 a}}{1 + e^{ik_1 a}} r_a = 1 + r_a$$

$$\implies \frac{\frac{k_2}{k_1}\left(\frac{1 - e^{ik_1 a}}{1 + e^{ik_1 a}}\right) - 1}{\frac{k_2}{k_1}\left(\frac{1 - e^{ik_1 a}}{1 + e^{ik_1 a}}\right) + 1} = r_a$$

Note $\quad \frac{\sin(k_1\frac{a}{2})}{\cos(k_1\frac{a}{2})} = \frac{e^{ik_1 a/2} - e^{-ik_1 a/2}}{i(e^{ik_1 a/2} + e^{-ik_1 a/2})} = \frac{e^{ik_1 a} - 1}{i(e^{ik_1 a} + 1)} = \left(\frac{1 - e^{ik_1 a}}{1 + e^{ik_1 a}}\right)(-i) = \tan\left(\frac{k_1 a}{2}\right)$

$$r_a = \frac{-i\frac{k_2}{k_1}\tan\left(\frac{k_1 a}{2}\right) - 1}{-i\frac{k_2}{k_1}\tan\left(\frac{k_1 a}{2}\right) + 1} \implies \boxed{r_a = -\frac{1 + i\frac{k_2}{k_1}\tan\left(\frac{k_1 a}{2}\right)}{1 - i\frac{k_2}{k_1}\tan\left(\frac{k_1 a}{2}\right)}}$$

Absolute value of r_a

$$|r_a| = \sqrt{r_a r_a^*} . \quad r_a r_a^* = \left(\frac{\cancel{1 + i\frac{k_2}{k_1}\tan(\,)}}{\cancel{1 - i\frac{k_2}{k_1}\tan(\,)}}\right)\left(\frac{\cancel{1 - i\frac{k_2}{k_1}\tan(\,)}}{\cancel{1 + i\frac{k_2}{k_1}\tan(\,)}}\right) = 1$$

$$\implies \boxed{|r_a| = 1}$$

100% of the radiation is reflected back due to the perfect conductor.