$^{+4}/_4$

$\boxed{38.2}$ Begin w/ $\dfrac{dP}{d\omega} = \dfrac{\omega^2 e^2}{2\pi c}\displaystyle\int_{-\infty}^{\infty} d\tau\, e^{-i\omega\tau}\left[\frac{\vec{v}(T+\tau/2)\cdot\vec{v}(T-\tau/2)}{c^2} - 1\right]\int_{-1}^{1} d\lambda\, e^{-i\frac{\omega}{c}\lambda|\vec{r}-\vec{r}'|}$

Our goal is to find P, so we integrate w.r.t. ω

$$P = \frac{e^2}{2\pi c}\int_{-\infty}^{\infty} d\omega\, \omega^2 \int_{-\infty}^{\infty} d\tau\, e^{-i\omega\tau}\left[\frac{\vec{v}\cdot\vec{v}'}{c^2} - 1\right]\int_{-1}^{1} d\lambda\, e^{-i\frac{\omega}{c}\lambda|\vec{r}-\vec{r}'|}$$

re-jigger the limits of integration:

$$P = \frac{e^2}{2\pi c}\int_{-\infty}^{\infty} d\tau\left(\int_{-1}^{1} d\lambda\left[\frac{\vec{v}\cdot\vec{v}'}{c^2} - 1\right]\int_{-\infty}^{\infty} d\omega\,\omega^2 e^{-i\omega\tau} e^{-i\frac{\omega}{c}\lambda|\vec{r}-\vec{r}'|}\right)$$

Let $z \equiv \tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|$

Then, the ω integral becomes $\int_{-\infty}^{\infty} d\omega\,\omega^2 e^{-i\omega z} = \dfrac{-d^2}{dz^2}\displaystyle\int_{-\infty}^{\infty} e^{-i\omega z}\, d\omega$

and, by 38.60, $= -2\pi\delta''(x)$

$= -2\pi\delta''(\tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|)$

$$P = \frac{e^2(-\cancel{2\pi})}{\cancel{2\pi} c}\int_{-\infty}^{\infty} d\tau\left(\int_{-1}^{1} d\lambda\left[\frac{\vec{v}\cdot\vec{v}'}{c^2} - 1\right]\delta''\left(\tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|\right)\right)$$

Multiply by $\frac{1}{2}$ b/c (evidently) the limits of integration in the book's equation run from $0 \to \infty$ instead of $-\infty \to \infty$, and the gaussian is even.

$$P = \frac{-e^2}{2c}\int_{-\infty}^{\infty} d\tau\int_{-1}^{1} d\lambda\left(\frac{\vec{v}\cdot\vec{v}'}{c^2} - 1\right)\delta''\left(\tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|\right)$$

Now, by 38.44, $|\vec{r}-\vec{r}'| = 2R|\sin(\frac{1}{2}\phi)|$ where $\phi \equiv \omega_0\tau$

Thus, $\delta''(\tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|) = \delta''(\tau + \frac{\lambda}{c}2R|\sin(\frac{\phi}{2})|)$

$= \delta''\left(\frac{1}{\omega_0}\left(\tau\omega_0 + 2\lambda\frac{R\omega_0}{c}\sin(\frac{\phi}{2})\right)\right)$

$R = \frac{v}{\omega_0}$, and $\beta = \frac{v}{c}$, so...

$= \delta''\left(\frac{1}{\omega_0}\left(\phi + 2\lambda\beta\sin(\frac{\phi}{2})\right)\right)$

Per 38.62, $y = \phi + 2\lambda\beta\sin(\frac{\phi}{2})$, so we have $\delta''(\frac{1}{\omega_0}y)$

Now, $\delta''(a\cdot x) = \frac{1}{a^3}\delta''(x)$, so finally, $\delta''(\tau + \frac{\lambda}{c}|\vec{r}-\vec{r}'|) = \omega_0^3\delta''(y)$

Next, $y = \omega_0\tau + 2\lambda\beta\sin\left(\frac{\omega_0\tau}{2}\right) \Rightarrow d\tau = \dfrac{dy}{\omega_0(1 + \frac{2\lambda}{2}\beta\cos(\frac{\phi}{2}))}$

$$\Rightarrow P = \frac{-e^2\omega_0^3}{2c\omega_0}\int_{-\infty}^{\infty}\frac{dy}{1+\lambda\beta\cos(\frac{\phi}{2})}\int_{-1}^{1} d\lambda\left[\frac{\vec{v}\cdot\vec{v}'}{c^2} - 1\right]\delta''(y)$$

Using the definition of the dot product, $\frac{\vec{v}\cdot\vec{v}'}{c^2} = \beta^2\cos(\phi)$

$$\Rightarrow P = \frac{-e^2\omega_0^2}{2c}\int_{-1}^{1} d\lambda\int_{-\infty}^{\infty} dy\,\frac{\beta^2\cos\phi - 1}{1+\lambda\beta\cos(\phi/2)}\,\delta''(y)$$

$$\Rightarrow \boxed{P = \frac{-e^2\beta}{2R}\int_{-1}^{1} d\lambda\int_{-\infty}^{\infty}\frac{\beta^2\cos\phi - 1}{1+\lambda\cos(\phi/2)}\,\delta''(y)}$$ which is 38.63. Hence 38.65.