Definitely playing it fast and loose w/ partials - be more careful next time

$\boxed{36.2}$

+3.5/4

by 36.19

$$-\frac{dE}{dz} = \int d\omega \cdot \omega \frac{e^2}{c^2}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) \quad \text{Differentiate both sides wrt } d\omega$$

$$-\frac{d^2E}{dz\,d\omega} = \omega\frac{e^2}{c^2}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) \qquad E=\hbar\omega \Rightarrow \frac{dE}{\hbar} = d\omega$$

$$\frac{-d^2E}{dz\,dE} = \frac{\omega}{\hbar}\frac{e^2}{c^2}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) = \omega\frac{\alpha}{c}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) \quad \text{where } \alpha = \frac{e^2}{\hbar c}$$

Total charge in this case is Ze, so $e \to Ze$.

$$\frac{-d^2E}{dE\,dz} = \frac{\omega\alpha Z^2}{c}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) \quad \text{also } E = N\hbar\omega \Rightarrow dE = dN\,\hbar\omega$$

$$\frac{-d^2N}{dE\,dz} = \frac{\cancel{\omega}\alpha Z^2}{\hbar\cancel{\omega}c}\left(1-\frac{c^2}{n^2(\omega)v^2}\right) = \frac{\alpha Z^2}{\hbar c}\left(1-\frac{c^2}{n^2(\omega)v^2}\right)$$

$$\sin^2\theta_c = 1-\cos^2\theta_c = \left(1-\frac{c^2}{n^2(\omega)v^2}\right) \text{ since } \cos\theta_c = \frac{c}{nv} \text{ by 36.15}$$

$$\boxed{\frac{d^2N}{dE\,dz} = \frac{\alpha Z^2}{\hbar c}\sin^2\theta_c}$$

when $Z=1$ and $\alpha = \frac{1}{137}$, $\hbar c = 2\cdot 10^{-5}\,\text{eV cm}$

$$\boxed{\frac{d^2N}{dE\,dz} = \frac{\alpha}{\hbar c}\sin^2\theta \approx 370\sin^2\theta_c(E)\,\text{eV}^{-1}\text{cm}^{-1}}$$

$$\text{Let } r_0 \equiv \frac{e^2}{m_e c^2} = \frac{\alpha\hbar c}{m_e c^2} \Rightarrow \alpha\hbar c = r_0 m_e c^2 \Rightarrow \hbar c = \frac{r_0 m_e c^2}{\alpha}$$

$$\Rightarrow \frac{dN}{dE\,dz} = \frac{\alpha^2 Z^2}{r_0 m_e c^2}\sin^2\theta_c = \frac{\alpha^2 Z^2}{r_0 m_e c^2}\left(1-\cos^2\theta_c\right) = \frac{\alpha^2 Z^2}{r_0 m_e c^2}\left(1-\frac{c^2}{v^2}\cdot\frac{1}{n^2}\right)$$

$$\text{Let } \beta = \frac{v}{c} \Rightarrow \boxed{\frac{d^2N}{dE\,dz} = \frac{\alpha^2 Z^2}{r_0 m_e c^2}\left(1-\frac{1}{\beta^2 n^2}\right)}$$

$$dE = -\frac{2\pi\hbar c}{\lambda^2}d\lambda$$

$$\text{Now, } E = hc/\lambda = 2\pi\hbar c/\lambda \Rightarrow \hbar c = \frac{E\lambda}{2\pi} \Rightarrow \frac{d^2N}{dE\,dz} = \frac{2\pi\alpha Z^2}{E\lambda}\left(1-\frac{1}{\beta^2 n^2(E)}\right)$$

$$\text{Finally, let } E\to\lambda: \boxed{\frac{d^2N}{d\lambda\,dz} = \frac{2\pi\alpha Z^2}{\lambda^2}\left(1-\frac{1}{\beta^2 n^2(\lambda)}\right)}$$

I'm surprised this worked...