$\boxed{35.2}$ Given: $\ddot{\vec{r}} = -\omega_0^2 \vec{r} - \gamma \dot{\vec{r}}$

This has exponential solutions. Assume $\vec{r} = \vec{a} e^{i\alpha t}$, where $\vec{a}$ is the displacement from the center, and α is a constant we must find. $\Rightarrow \dot{\vec{r}} = i\alpha \vec{a} e^{i\alpha t}$, $\ddot{\vec{r}} = -\alpha^2 \vec{a} e^{i\alpha t}$

Plug these into the given ODE:

$$-\alpha^2 \vec{a} \cancel{e^{i\alpha t}} + \gamma i \alpha \vec{a} \cancel{e^{i\alpha t}} + \omega_0^2 \vec{a} \cancel{e^{i\alpha t}} = 0$$

$$-\alpha^2 + \gamma i \alpha + \omega_0^2 = 0$$

Solve for α (quadratic equation):

$$\alpha = \frac{i\gamma}{2} \pm \frac{\sqrt{4\omega_0^2 - \gamma^2}}{2} = \frac{i\gamma}{2} \pm \omega_0 \sqrt{1 - \frac{\gamma^2}{4\omega_0^2}}$$

$$\therefore \vec{r}(t) = \vec{a} \exp\left(i\left(\frac{i\gamma}{2} \pm \omega_0\sqrt{1 - \frac{\gamma^2}{4\omega_0^2}}\right)t\right) = \vec{a} e^{-\gamma t/2} e^{\pm i\omega_0 t\sqrt{1-\gamma^2/4\omega_0^2}}$$

Let $\tilde{\omega}_0 \equiv \omega_0 \sqrt{1 - \gamma^2/4\omega_0^2}$

$$\vec{r}(t) = \vec{a} e^{-\gamma t/2} e^{\pm i t \tilde{\omega}_0} = \vec{a} e^{-\gamma t/2}\left(\cos(\tilde{\omega}_0 t) \pm i\sin(\tilde{\omega}_0 t)\right)$$

But position should not be imaginary, so get rid of the sine term.

$$\boxed{\vec{r}(t) = \vec{a} e^{-\gamma t/2} \cos(\tilde{\omega}_0 t) \qquad t > 0}$$ (otherwise it would blow up!)

$$V(t) = \frac{d\vec{r}}{dt} = \frac{\vec{a}\gamma}{2} e^{-\gamma t/2}\cos(\tilde{\omega}_0 t) - \tilde{\omega}_0 \vec{a} e^{-\gamma t/2}\sin(\tilde{\omega}_0 t)$$

Fourier transform:

$$\vec{V}(\omega) = \int_0^\infty dt\, e^{i\omega t}\, \vec{V}(t) = \frac{\vec{a}\gamma}{4}\int_0^\infty dt\, e^{(i\omega t - \gamma t/2)}\left(e^{i\tilde{\omega}_0 t} - e^{-i\tilde{\omega}_0 t}\right)$$

$$- \frac{\tilde{\omega}_0 \vec{a} i}{2}\int_0^\infty dt\, e^{(i\omega t - \gamma t/2)}\left(e^{i\tilde{\omega}_0 t} - e^{-i\tilde{\omega}_0 t}\right)$$

$$= \frac{\vec{a}\gamma}{4}\int_0^\infty dt\, e^{t(i\omega - \frac{\gamma}{2} + i\tilde{\omega}_0)} + \frac{a\gamma}{4}\int_0^\infty dt\, e^{t(i\omega - \frac{\gamma}{2} - i\tilde{\omega}_0)}$$

$$- \frac{i\tilde{\omega}_0 a}{2}\int_0^\infty dt\, e^{t(i\omega - \frac{\gamma}{2} + i\tilde{\omega}_0)} + \frac{i\tilde{\omega}_0 \vec{a}}{2}\int_0^\infty dt\, e^{t(i\omega - \gamma/2 - i\tilde{\omega}_0)}$$

$$= \left(\frac{\vec{a}\gamma}{4} - \frac{i\tilde{\omega}_0 \vec{a}}{2}\right)\int_0^\infty e^{t(i\omega - \frac{\gamma}{2} + i\omega_0)} + \left(\frac{\vec{a}\gamma}{4} + \frac{i\tilde{\omega}_0 \vec{a}}{2}\right)\int_0^\infty dt\, e^{t(i\omega - \gamma/2 - i\tilde{\omega}_0)}$$

$$= \frac{\vec{a}\left(\frac{\gamma}{2} - \frac{i\tilde{\omega}_0}{2}\right)}{i\omega - \gamma/2 + i\tilde{\omega}_0} + \frac{\vec{a}\left(\frac{\gamma}{4} + \frac{i\tilde{\omega}_0}{2}\right)}{i\omega - \gamma/2 - i\tilde{\omega}_0}$$

$$\therefore \boxed{\vec{V}(\omega) = \frac{i\vec{a}}{2}\left[\frac{-\tilde{\omega}_0 - i\gamma/2}{i(\omega + \omega_0 + i\gamma/2)} + \frac{\tilde{\omega}_0 - \frac{i\gamma}{2}}{i(\omega - \tilde{\omega}_0 + \frac{i\gamma}{2})}\right]}$$