+3/3

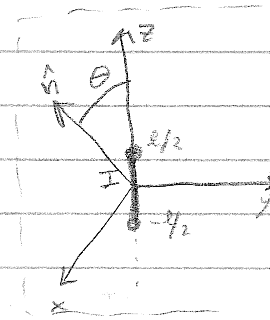
$\boxed{34.1}$ In order to derive $\frac{dP}{d\Omega}$ for a short antenna, we begin with equation 32.29, $\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}(\hat{n} \times \ddot{\vec{p}})^2$

We begin with $\ddot{\vec{p}} = \frac{d^2}{dt^2}\vec{p} = \frac{\partial^2}{\partial t^2}\int d\vec{r}\, \rho(\vec{r},t)\,\vec{r}$ ✓

Now note that $\frac{\partial}{\partial t}\int d\vec{r}\, \rho(\vec{r},t)\,\vec{r} \approx \frac{\partial}{\partial t}\sum_k q_k \vec{r}_k(t) = \sum_k q_k \vec{v}_k(t) \approx \int d\vec{r}\, \vec{j}(\vec{r},t)$

$\therefore \ddot{\vec{p}} = \frac{\partial}{\partial t}\int d\vec{r}\, \vec{j}(\vec{r},t) = \int d\vec{r}\, \frac{\partial}{\partial t}\vec{j}(\vec{r},t)$ ✓

Arrange the antenna as shown to the right with current I, and length ℓ along the z-axis.



Then, $\vec{j}(\vec{r},t) = J_z\hat{z} = I\,\delta(x)\,\delta(y)\sin(\omega t)$

(assuming the frequency of oscillation is ω)

Now

✓ $\ddot{\vec{p}} = I\hat{z}\int_{-\ell/2}^{\ell/2} \frac{d}{dt}\sin(\omega t)\,dz = I\ell\omega\cos(\omega t)\,\hat{z}$

Plug $\ddot{\vec{p}}$ into 32.29: $\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}(\hat{n} \times I\ell\omega\cos(\omega t)\,\hat{z})^2$

$$= \frac{I^2\ell^2\omega^2\cos^2(\omega t)}{4\pi c^3}(\hat{n}\times\hat{z})^2$$

$\hat{n}\times\hat{z} = \sin\theta$, when $\hat{n}$ is the direction of observation.

✓ $$\frac{dP}{d\Omega} = \frac{I^2\ell^2\omega^2}{4\pi c^3}\sin^2\theta\cos^2(\omega t)$$ ✓

average $\cos^2(\omega t)$ over a period. ✓ $\overline{\cos^2(\omega t)} = \frac{1}{2}$

$\therefore \frac{dP}{d\Omega} = \frac{I^2\ell^2\omega^2}{4\pi c^3}\sin^2\theta$. ✓

finally, notice that $\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \Rightarrow \omega^2 = \frac{4\pi^2 c^2}{\lambda^2} \Rightarrow \frac{\omega^2}{4c^2} = \frac{\pi^2}{\lambda^2}$

$$\boxed{\frac{dP}{d\Omega} = \frac{I^2}{2\pi c}\left(\frac{\ell\pi}{\lambda}\right)^2\sin^2\theta}$$