

32.1 we begin with the Larmor formula for the total power radiated by an accelerated charged particle of charge  $e$ : (32.25)

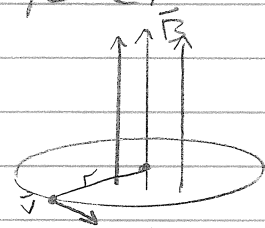
$$P = \frac{2e^2}{3c^3} a^2$$

For the orbiting particle,  $a = \frac{v^2}{r}$

Also, since  $\vec{F} = m\vec{a} = \frac{e}{c}(\vec{v} \times \vec{B})$ ,

$\vec{a} = \frac{e v B}{m c}$  (for the circular orbit where  $|\vec{v} \times \vec{B}| = vB$ .)

$$\Rightarrow a = \frac{v^2}{r} = \frac{e v B}{m c} \Rightarrow a = \frac{e v B}{m c}$$



Plug this into the Larmor formula:  $P = \frac{2e^4 v^2 B^2}{3c^5 m^2}$

The Energy of the particle is  $E = \frac{1}{2} m v^2$ ,

$$P = \frac{2e^4 v^2 B^2}{3c^5 m^2} \left( \frac{E \cdot 2}{m v^2} \right) = \frac{4e^4 B^2}{3c^5 m^3} E = -\frac{dE}{dt}$$

So we set

$$\tau = \frac{4e^4 B^2}{3c^5 m^3}$$

$$\text{and } P = \tau E = -\frac{dE}{dt}$$

The solution to the ODE is  $E(t) = E(0) e^{-t/\tau}$ , where

$1/\tau = \frac{3c^5 m^3}{4e^4 B^2}$  is the mean lifetime of motion.

Lets convert this to SI units.  $B \rightarrow \sqrt{\frac{4\pi}{260}} B$  |  $e \rightarrow \frac{1}{\sqrt{4\pi\epsilon_0}} e$

$$\therefore \frac{1}{\tau} = \frac{3c^5 m^3}{4e^4 B^2} \frac{\mu_0}{4\pi} (4\pi\epsilon_0)^2 = \frac{3c^5 m^3}{4e^4 B^2} \mu_0 4\pi\epsilon_0^2 = \frac{3\pi c^3 m^3}{e^4 B^2} \epsilon_0^2$$

Now plug in the given values ( $B = 10^4 \text{ g} = 1 \text{ T}$ )

The calculation is done in the attached Mathematica document:

$$\frac{1}{\tau} = 2.6 \text{ sec}^{-1}$$