$\boxed{31.1}$ Begin by noting that $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$, and $\vec{r}(t') = Vt'$, so

$$\vec{r} - \vec{r}(t') = x\hat{x} + y\hat{y} + (z - Vt')\hat{z} \quad \text{and} \quad |\vec{r} - \vec{r}(t')| = \sqrt{x^2+y^2+(z-Vt')^2}$$

Also see that $\vec{V}(t') = V\hat{z}$, so $(\vec{r} - \vec{r}(t')) \cdot \frac{\vec{V}(t')}{c} = \frac{V}{c}(z - Vt')$

Then $|\vec{r} - \vec{r}(t')| - (\vec{r} - \vec{r}(t')) \cdot \frac{\vec{V}(t')}{c} = \sqrt{x^2+y^2+(z-Vt')^2} - \frac{V}{c}(z - Vt') \equiv \underline{\text{denom}}$

$$= \sqrt{x^2+y^2+(z-Vt')^2} - \frac{V}{c}z + \frac{V^2 t}{c} - \frac{V^2}{c^2}\sqrt{x^2+y^2+(z-Vt')^2} \quad \left(\text{b/c } t' = t - \frac{1}{c}\sqrt{x^2+y^2+(z-Vt')^2}\right.$$

$$= \left(1 - \frac{V^2}{c^2}\right)\sqrt{x^2+y^2+(z-Vt')^2} + \frac{V}{c}(Vt - z)$$

I don't know. I keep trying... and can't get it to work.

$$= \sqrt{(z-Vt)^2 + \left(1 - \frac{V^2}{c^2}\right)(x^2+y^2)}$$

So we replace the denominator in our potential from 31.5 and get

$$\boxed{\Phi = \frac{e}{\sqrt{(z-Vt)^2 + \left(1 + \frac{V^2}{c^2}\right)(x^2+y^2)}}}$$

In part 31.5, we saw $\vec{A} = \frac{V(t')}{c}\Phi(\vec{r}, t)$, and now since $\vec{V}(t) \to \vec{V}$

$$\boxed{\vec{A} = \frac{\vec{V}}{c}\Phi}$$

Electric field

$$\vec{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A} = -\nabla\frac{e}{\sqrt{(z-vt)^2 + (1-\frac{v^2}{c^2})(x^2+y^2)}} - \frac{e}{c^2}\frac{\partial}{\partial t}\frac{\vec{v}}{\sqrt{(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)}}$$

$$= -e\,\frac{-\frac{1}{2}(1+\frac{v^2}{c^2})(2x\hat{x}+2y\hat{y}) + -\frac{1}{2}\,2(z-vt)\cdot 1}{\left[(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)\right]^{3/2}} - \frac{e\vec{v}}{c^2}\,\frac{-\frac{1}{2}\,2(z-vt)(-v)}{[\ldots\ldots\ldots]^{3/2}}$$

$$= e\,\frac{(1-\frac{v^2}{c^2})(x\hat{x}+y\hat{y}) + (z-vt)}{[\ldots\ldots\ldots]^{3/2}} - \frac{e\vec{v}v}{c^2}\,\frac{z-vt}{[\ldots\ldots\ldots]^{3/2}}$$

$$= e\,\frac{(1-\frac{v^2}{c^2})(x\hat{x}+y\hat{y}) + (z-vt) - \frac{v^2}{c^2}(z-vt)\hat{z}}{[\ldots\ldots\ldots]^{3/2}} \qquad (\text{b/c } \vec{v} = v\hat{z})$$

$$= e\,\frac{(1-\frac{v^2}{c^2})(x\hat{x}+y\hat{y}+(z-vt)\hat{z})}{[\ldots\ldots\ldots]^{3/2}}$$

$$\boxed{\vec{E} = e\,\frac{(1-\frac{v^2}{c^2})(\vec{r}-\vec{v}t)}{\left[(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)\right]^{3/2}}}$$

Magnetic Field

$$\vec{B} = \vec{\nabla}\times\vec{A} = \vec{\nabla}\times\frac{e\vec{v}}{c\sqrt{(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)}}$$

To mathematica! (see attached)

$$\vec{B} = \frac{ev(1-\frac{v^2}{c^2})(y\hat{x}+x\hat{y})}{c[\ldots\ldots\ldots]^{3/2}}$$

Same as before

Now, note $\hat{z}\times(\vec{r}-\vec{v}t) = \hat{z}\times(x\hat{x}+y\hat{y}+(z-vt)\hat{z}) = y\hat{x}+x\hat{y}$,

so

$$\vec{B} = \frac{ev(1-\frac{v^2}{c^2})(\hat{z}\times(\vec{r}-\vec{v}t))}{c\left[(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)\right]^{3/2}} = \frac{v\hat{z}}{c}\times\underbrace{\frac{e(1-\frac{v^2}{c^2})(\vec{r}-\vec{v}t)}{\left[(z-vt)^2+(1-\frac{v^2}{c^2})(x^2+y^2)\right]^{3/2}}}_{E}$$

$$\boxed{\vec{B} = \frac{\vec{v}}{c}\times\vec{E}}$$