$\checkmark$ 3.7 Given $\vec{F} = \vec{E} + i\vec{B}$, $\vec{F}^* = \vec{E} - i\vec{B}$

3/3

Now I'm going to calculate the following 3 quantities:

1) $\frac{1}{8\pi}\vec{F}^*\vec{F} = \frac{1}{8\pi}\left[(\vec{E} - i\vec{B})(\vec{E} + i\vec{B})\right] = \frac{1}{8\pi}(E^2 + B^2) = \boxed{u}$ energy density

2) $\frac{1}{8\pi i}\vec{F}^* \times \vec{F} = \frac{1}{8\pi i}(\vec{E} - i\vec{B}) \times (\vec{E} + i\vec{B}) = \frac{1}{8\pi i}\left(\cancel{\vec{E}\times\vec{E}} - i\vec{B}\times\vec{E} + \vec{E}\times i\vec{B} + \cancel{\vec{B}\times\vec{B}}\right)$

$= \frac{1}{8\pi}\left(\vec{E}\times\vec{B} + \vec{E}\times\vec{B}\right) = \frac{1}{4\pi}(\vec{E}\times\vec{B}) = \boxed{\vec{S}/c}$ energy flux vector $/c$

3) $\frac{1}{8\pi}(\vec{F}\vec{F}^* + \vec{F}^*\vec{F}) = \frac{1}{8\pi}\left((\vec{E} + i\vec{B})(\vec{E} - i\vec{B}) + (\vec{E} - i\vec{B})(\vec{E} + i\vec{B})\right)$

$= \frac{1}{8\pi}\left(\vec{E}\vec{E} + \cancel{i\vec{B}\vec{E}} - \cancel{\vec{E}i\vec{B}} + \vec{B}\vec{B} + \vec{E}\vec{E} - \cancel{i\vec{B}\vec{E}} + \cancel{\vec{E}i\vec{B}} + \vec{B}\vec{B}\right)$

$= \frac{1}{4\pi}\left(\vec{E}\vec{E} + \vec{B}\vec{B}\right) = -\overleftrightarrow{T} + \hat{\mathbb{1}}\,\frac{E^2 + B^2}{8\pi}$

$= \boxed{\hat{\mathbb{1}}u - \overleftrightarrow{T}}$ (by 3.14)

Now let $\vec{F} \to e^{-i\phi}\vec{F}$ (which implies $\vec{F}^* \to e^{+i\phi}\vec{F}$)

1) $\frac{1}{8\pi}\vec{F}^*\vec{F} \longrightarrow \frac{1}{8\pi}\cancel{e^{i\phi}e^{-i\phi}}\vec{F}^*\vec{F} = u$

2) $\frac{1}{8\pi i}\vec{F}^*\times\vec{F} \longrightarrow \frac{1}{8\pi}\cancel{e^{i\phi}e^{-i\phi}}(\vec{F}^*\times\vec{F}) = \vec{S}/c$

3) $\frac{1}{8\pi}(\vec{F}\vec{F}^* + \vec{F}^*\vec{F}) \longrightarrow \frac{1}{8\pi}e^{i\phi}e^{-i\phi}(\vec{F}\vec{F}^* + \vec{F}^*\vec{F}) = \boxed{\hat{\mathbb{1}}u - \overleftrightarrow{T}}$

<u>All as before</u>

Thus u, S, and $\hat{\mathbb{1}}u - \overleftrightarrow{T}$, and thus $\overleftrightarrow{T}$ are all invariant under duality.