+3/3

#2 Schwinger Problem 28.2

Ok. There is a much easier way to do this. I get it now. You just use the B.C. $\hat{r}\cdot\vec{B}\big|_{r=a}=0$ Plug in B_{tot}. Solve for μ. wow...

Nice work, anyway

We begin by defining the unit vectors in spherical coordinates.

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In[419]:= R = {1, 0, 0};
          Θ = {0, 1, 0};
          Φ = {0, 0, 1};
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Then we define the unit vector in the z direction in terms of spherical unit vectors

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Z = Cos[θ] R - Sin[θ] Θ;
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There are two contributions to the magnetic field. One is from the constant b-field (B $\hat{z}$), while the other is from the induced dipole moment on the sphere. We are given the equation for the magnetic field due to an electric dipole moment in Equation 28.16. Assuming that μ is in the $\hat{z}$ direction, and cancelling two factors of r, we can write down the total magnetic field:

In[422]:= $\text{Btot} = \text{B Z} + \dfrac{3\,\text{Dot}[\mu\,\text{Z},\ \text{R}]\ \text{R} - \mu\,\text{Z}}{\text{r}^3};$

Now, notice that the sphere shields its interior from magnetic field. Thus there is a discontinuity between the magnetic field on either side of the boundary shell. That gives rise to a surface current density $\vec{K}$. In general, we can find the current density using Gaussian units as follows:

$\vec{K} = \frac{c}{4\pi}\hat{n}\times\left(\vec{B}_2-\vec{B}_1\right)_{r=a} = \frac{c}{4\pi}\left(\hat{r}\times\vec{B}_{\text{tot}}\right)_{r=a}$ in this case.

In order to find the magnetic moment, we carry out the following surface integral

$\vec{\mu} = \frac{1}{2c}\int\vec{r}\times\vec{K}\,dS$

To this end, let's calculate $\vec{r}\times\vec{K}$.

In[430]:= $\text{rxK} = \text{Cross}\left[\text{r R},\ \dfrac{\text{c}}{4\pi}\text{Cross}[\text{R},\ \text{Btot}]\right]\ /.\ \text{r}\to\text{a}$

Out[430]= $\left\{0,\ \dfrac{\text{a B c Sin}[\theta]}{4\pi} - \dfrac{\text{c}\,\mu\,\text{Sin}[\theta]}{4\,\text{a}^2\,\pi},\ 0\right\}$

Thus, $\vec{r}\times\vec{K} = \left(\frac{\text{a B c Sin}[\theta]}{4\pi} - \frac{\text{c}\,\mu\,\text{Sin}[\theta]}{4\,\text{a}^2\,\pi}\right)\hat{\theta}$. Now, we want to calculate the surface integral over a spherical shell. The only part of this result that will contribute to the surface integral will be the z-component, since all the x and y components will obviously cancel themselves out.

To get the z-projection, we multiply $\vec{r}\times\vec{K}$ by -Sin[θ]. (that's because $\hat{z}=\text{Cos}\theta\,\hat{r}-\text{Sin}\theta\,\hat{\theta}$). Thus, the relevant portion of our integrand is as follows. (not it's in the z-direction now!).

In[431]:= $\text{Integrand} = -\text{Sin}[\theta]\left(\dfrac{\text{a B c Sin}[\theta]}{4\pi} - \dfrac{\text{c}\,\mu\,\text{Sin}[\theta]}{4\,\text{a}^2\,\pi}\right)\ \text{// FullSimplify}$

Out[431]= $\dfrac{\text{c}\left(-\text{a}^3\,\text{B}+\mu\right)\text{Sin}[\theta]^2}{4\,\text{a}^2\,\pi}$

Now just take the surface integral
$\vec{\mu} = \frac{1}{2c}\int_0^{\pi}\int_0^{2\pi}\text{Integrand}\, a^2 \,\text{Sin}\theta \, d\phi \, d\theta \, \hat{z}$

In[437]:=
$$\text{mu} = \frac{1}{2\,c}\int_0^{\pi}\int_0^{2\pi}\text{Integrand}\, a^2\, \text{Sin}[\theta]\, d\phi\, d\theta \; // \; \text{FullSimplify}$$

Out[437]=
$$\frac{1}{3}\left(-a^3\, B + \mu\right)$$

Then all that's left is to solve for μ:

In[438]:=
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Solve[μ == mu, μ]
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Out[438]=
$$\left\{\left\{\mu \to -\frac{a^3\, B}{2}\right\}\right\}$$

As expected, we got $\vec{\mu} = -\frac{1}{2}a^3 B\, \hat{z}$