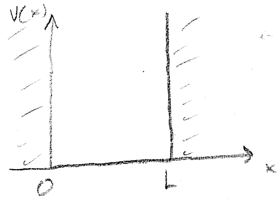
7.9



(a) By 7.3.4, the triplet state is symmetrical

For two electrons, we are given 7.3.14

$\phi = \frac{1}{\sqrt{2}}\left(\omega_A(x_1)\,\omega_B(x_2) \pm \omega_A(x_2)\,\omega_B(x_1)\right)$

Fermions must be overall antisymmetric, and since we saw that the spin part of the wavefunction is symmetric, we choose the bottom sign.

First note that for a Square well, $\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$, $\psi_1(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$

$$\Rightarrow \boxed{\phi_{g.s.,\,\text{triplet}}(x_1,x_2) = \frac{\sqrt{2}}{L}\left(\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right)}$$

where I let $\omega_A \to \psi_1$ and $\omega_B \to \psi_2$

also, for a Square well, $E_n = \frac{n^2\pi^2\hbar^2}{2ml^2} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2mL^2}$, $E_2 = \frac{2\pi^2\hbar^2}{mL^2}$

$$\Rightarrow E = E_1 + E_2 = \boxed{\frac{5\pi^2\hbar^2}{2mL^2}}$$

(b) The singlet state, in contrast, is completely antisymmetric. Thus, the spatial part of the wavefunction must be symmetric since these are fermions. We choose the TOP sign of 7.3.14, and note that now the symmetry allows two fermions to fall into the ground state

$$\phi_{g.s.,\,\text{singlet}} = \frac{\sqrt{2}}{L}\left(\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right) + \sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{\pi x_1}{L}\right)\right)$$

$$\boxed{\phi_{g.s.,\,\text{singlet}} = \frac{2\sqrt{2}}{L}\left(\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right)}$$

energy?

(c) $V = -\lambda\delta(x_1 - x_2)$, $(\lambda > 0)$

Let's use 1st-order perturbation theory to find $\Delta^{(1)}_{\text{singlet}}$, and $\Delta^{(1)}_{\text{triplet}}$

$$\Delta^{(1)}_{\text{triplet}} = \langle \phi_{g.s.\,\text{triplet}} | V | \phi_{g.s.\,\text{triplet}} \rangle$$

$$= \frac{\sqrt{2}}{L}\int_0^L\int_0^L dx_1\,dx_2\left(\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)\right)^2(-\lambda\delta(x_1-x_2))$$

$$= \sqrt{2}\int_0^L dx\left(\cancel{\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right) - \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)}\right)^2(-\lambda) = 0$$

There is no change in the triplet state to first order.

$$\Delta^{(1)}_{\text{singlet}} = \langle \phi_{g.s.\,\text{singlet}} | V | \phi_{g.s.\,\text{singlet}} \rangle = -\lambda\frac{6}{L^2}\int_0^L\int_0^L\left(\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right)^2\delta(x_1-x_2)\,dx_1\,dx_2$$

$$= -\lambda\frac{6}{L^2}\int_0^L \sin^4\left(\frac{\pi x}{L}\right)dx = -\lambda\frac{6}{L^2}\frac{3L}{8} = \boxed{\frac{-9\lambda}{24L}}$$

So the potential causes an energy shift to the para state, but not the ortho state

96/100

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