$\boxed{7.3}$

For 2 spin 1 particles, we can have $j = 0, 1, 2$. Let's practice....

<u>$j=2$ quintet</u>

ⓐ $\rightarrow |2\,2\rangle = |1\,1;1\,1\rangle$

$J_-|2\,2\rangle = (J_{1-}+J_{2-})|1\,1;1\,1\rangle \Rightarrow 2\hbar|2,1\rangle = \hbar\sqrt{2}|1\,1;0\,1\rangle + \hbar\sqrt{2}|1\,1;1\,0\rangle$

(using Zettilli 5.56)

ⓑ $\rightarrow |2\,1\rangle = \frac{1}{\sqrt{2}}\left(|1\,1,0\,1\rangle + |1\,1,1\,0\rangle\right)$

$J_-|2\,1\rangle = \frac{1}{\sqrt{2}}\left((J_{1-}+J_{2-})|1\,1\,0\,1\rangle + (J_{1-}+J_{2-})|1\,1\,1\,0\rangle\right)$

$\Rightarrow \hbar\sqrt{6}|2\,0\rangle = \frac{1}{\sqrt{2}}\left(\hbar\sqrt{2}|1\,1\,-1\,1\rangle + \hbar\sqrt{2}|1\,1\,0\,0\rangle + \hbar\sqrt{2}|1\,1\,0\,0\rangle + \hbar\sqrt{2}|1\,1\,1\,-1\rangle\right)$

ⓒ $\rightarrow |2\,0\rangle = \frac{1}{\sqrt{6}}\left(2|1\,1,0\,0\rangle + |1\,1,-1\,1\rangle + |1\,1,1\,-1\rangle\right)$

etc.

ⓓ $\rightarrow |2\,-1\rangle = \frac{1}{\sqrt{2}}\left(|1\,1,0\,-1\rangle + |1\,1,-1\,0\rangle\right)$

ⓔ $\rightarrow |2,-2\rangle = |1\,1,-1\,-1\rangle$

<u>$j=1$ triplet</u>

$\rightarrow |1\,1\rangle = A|1\,1;1\,0\rangle + B|1\,1;0\,1\rangle$

Conditions: $\langle 1\,1|1\,1\rangle = 1 = A^2+B^2 \Rightarrow$

$\langle 2\,1|1\,1\rangle = 0 = \frac{A}{\sqrt{2}} + \frac{B}{\sqrt{2}} \Rightarrow \quad A = -B$ and $A = \frac{1}{\sqrt{2}}$

ⓕ $\rightarrow |1\,1\rangle = \frac{1}{\sqrt{2}}|1\,1;1\,0\rangle - \frac{1}{\sqrt{2}}|1\,1;0\,1\rangle$

$J_-|1\,1\rangle = \frac{1}{\sqrt{2}}\left((J_{1-}+J_{2-})|1\,1;1\,0\rangle - (J_{1-}+J_{2-})|1\,1;0\,1\rangle\right)$

$\hbar\sqrt{2}|1,0\rangle = \frac{1}{\sqrt{2}}\left(+\hbar\sqrt{2}|1\,1\,-1\,1\rangle + \hbar\sqrt{2}|1\,1\,0\,0\rangle - \hbar\sqrt{2}|1\,1\,0\,0\rangle - \hbar\sqrt{2}|1\,1\,1\,-1\rangle\right)$

ⓖ $\rightarrow |1\,0\rangle = \frac{1}{\sqrt{2}}\left(|1\,1,-1\,1\rangle - |1\,1,1\,-1\rangle\right)$

etc

ⓗ $\rightarrow |1\,-1\rangle = \frac{1}{\sqrt{2}}\left(|1\,1,0\,-1\rangle - |1\,1,-1\,0\rangle\right)$

<u>$j=0$ singlet</u>

$|0\,0\rangle = A|1\,1\,0\,0\rangle + B|1\,1\,-1\,1\rangle + C|1\,1,1\,-1\rangle$

conditions: $\langle 0\,0|0\,0\rangle = 1 \Rightarrow A^2+B^2+C^2 = 1$

$\langle 2\,0|0\,0\rangle = 0 \Rightarrow \quad 0 = \frac{2}{\sqrt{6}}A + \frac{1}{\sqrt{6}}B + \frac{1}{\sqrt{6}}C$

$\langle 1\,0|0\,0\rangle = 0 \Rightarrow \quad 0 = \frac{B}{\sqrt{2}} - \frac{C}{\sqrt{2}}$

$\Rightarrow A = \frac{1}{\sqrt{3}}, \quad B = C = \frac{-1}{\sqrt{3}}$

ⓘ $\rightarrow |0\,0\rangle = \frac{1}{\sqrt{3}}\left(|1\,1,0\,0\rangle - |1\,1,-1\,1\rangle - |1\,1,1\,-1\rangle\right)$

These are Bosons, so each state must be symetric. That is, if we flip $m_1 \leftrightarrow m_2$, we must get the same state. States a, b, c, d, e, and i satisfy this.

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$\boxed{\text{Restriction: } j \neq 1}$