6.7

(a) Our goal is to find δ_0.

we know that the wavefunction must satisfy the infinite potential boundary condition at $r=a$. thus, by 6.4.52

$$0 = e^{i\delta_0}[\cos\delta_0 \, j_0(ka) - \sin\delta_0 \, n_0(ka))$$

Note $j_0(ka) = \dfrac{\sin(ka)}{ka}$, $n_0(ka) = \dfrac{-\cos(ka)}{ka}$

$$\Rightarrow \quad 0 = \frac{\cos\delta_0 \sin(ka)}{ka} + \frac{\sin\delta_0 \cos(ka)}{ka}$$

$$-\sin\delta_0 \cos(ka) = \cos\delta_0 \sin(ka) \longrightarrow -\tan(\delta_0) = \tan(ka)$$

$$\Rightarrow \boxed{\delta_0 = -ka}$$

(b) The expansion of $f(\theta)$ is given in Legendre Polynomials;

$$f(\theta) = \frac{1}{k} \sum_l (2l+1) e^{i\delta_l} \sin\delta_l \, P_l(\cos\theta)$$

In the low-energy limit, $l=0$ suffices

$$f(\theta) = \frac{1}{k} e^{i\delta_0} \sin\delta_0 \, P_0(\cos\theta) = -\frac{1}{k} e^{-ika} \sin(ka)$$

where in the last step I plugged in $\delta_0 = -ka$ from part (a).

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{\sin^2(ka)}{k^2}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{\sin^2(ka)}{k^2} \int_0^{\pi} \int_0^{2\pi} d\phi \sin\theta \, d\theta = \boxed{\frac{4\pi \sin^2(ka)}{k^2}}$$

In the limit where $k \to 0$, $\sigma \to 4\pi a^2$, which is 4x the size of the geometric cross section. Quantum tells us that the radiation can bend around the sphere.