$\boxed{6.2}$ ⓐ By 6.2.28, $\frac{d\sigma}{d\Omega} = |f(\vec{k}', \vec{k})|^2$

The First Born Approx: $f^{(1)}(\vec{k}', \vec{k}) = \frac{-m}{2\pi\hbar^2}\int d^3x' \, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}'} V(x')$

Putting these together, we have

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2\hbar^4}\left|\int d^3x' V(x') e^{i(\vec{k}-\vec{k}')\cdot\vec{x}'}\right|^2$$

Expanding, and using double-primes for book-keeping. regroup the k's and x's like so

$$\Rightarrow \sigma = \frac{m^2}{4\pi^2\hbar^4}\iint d\Omega_k \, d\Omega_{k'} \int d^3x' d^3x'' \, V(x')V(x'') \, e^{-i\vec{k}(\vec{x}'-\vec{x}'')} e^{i\vec{k}'(\vec{x}''-\vec{x}')}$$

$$= \frac{m^2}{4\pi^2\hbar^4}\iint d^3x' d^3x'' \, V(x')V(x'') \int d\Omega_k \, e^{-i\vec{k}(\vec{x}'-\vec{x}'')} \int d\Omega_{k'} \, e^{-i\vec{k}'(\vec{x}'-\vec{x}'')}$$

Convert to spherical coords.

Now note that $\vec{k}\cdot(\vec{x}'-\vec{x}'') = k|\vec{x}'-\vec{x}''|\cos\theta$

Then the integals becomes easy. e.g:

$$\int d\Omega_k \, e^{-ik|\vec{x}'-\vec{x}''|\cos\theta} = \int_{-1}^{1}\int_0^{2\pi} d\phi \, d(\cos\theta) \, e^{-ik|\vec{x}'-\vec{x}''|\cos\theta} = \frac{4\pi \sin(k|\vec{x}'-\vec{x}''|)}{k|\vec{x}'-\vec{x}''|}$$

$$\therefore \sigma = \frac{m^2}{4\pi^2\hbar^4}\iint d^3x' d^3x'' \, V(r')V(r'') \, 4\pi \frac{\sin(k|\vec{x}'-\vec{x}''|)}{k|\vec{x}'-\vec{x}''|} \, \frac{\sin(k|\vec{x}'-\vec{x}''|)}{k|\vec{x}'-\vec{x}''|}$$

note $|\vec{k}'| = |\vec{k}|$

Simplify, and set $\vec{x}' \to \vec{x}$, $\vec{x}'' \to \vec{x}'$

$$\boxed{\sigma = \frac{m^2}{\pi\hbar^4}\iint d^3x \, d^3x' \, V(r)V(r') \frac{\sin^2(k|\vec{x}-\vec{x}'|)}{k^2|\vec{x}-\vec{x}'|^2}}$$

ⓑ Optical Theorem: $\sigma = \frac{4\pi}{k}\,\mathrm{Im}\, f^{(2)}(\theta=0)$

By 6.3.17, $f^{(2)} = -\frac{1}{4\pi}\left(\frac{2m}{\hbar^2}\right)\int d^3x' \int d^3x'' \, e^{-i\vec{k}\cdot\vec{x}'} V(\vec{x}')V(\vec{x}'') \, e^{i\vec{k}\cdot\vec{x}''}\left(\frac{2m}{\hbar^2} G_+\right)$

By 6.2.11, $G_+ = -\frac{1}{4\pi}\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$

The imaginary par of G_+ is $-\frac{1}{4\pi}\frac{\sin(k|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}$ Let's just plug that in now b/c ultimaly we take $\mathrm{Im}(f^{(2)})$

$$\sigma = \mathrm{Im}\left[\frac{4m^2}{16\pi^2\hbar^4}\iint d^3x' d^3x'' \, V(r')V(r'') \, e^{-ik\cdot(\vec{x}'-\vec{x}'')} \frac{\sin(k|\vec{x}-\vec{x}'|)}{k|\vec{x}-\vec{x}'|}\right]\frac{4\pi}{k}$$

$$\boxed{\sigma = \frac{m^2}{\pi\hbar^4}\iint d^3x \, d^3x' \, V(r)V(r') \frac{\sin^2(k|\vec{x}-\vec{x}'|)}{k^2|\vec{x}-\vec{x}'|}}$$

not sure why it's not squared..