$\boxed{5.7}$ In gaussian units, the portaial of a constat E-field in the z-direction is $\frac{q}{z} = \Phi$

Thus, since $q = e$, and $|\vec{E}| = E = \frac{q}{z^2}$, $\Phi = zE$

$U = -q \cdot \Phi$, so $U = -zeE$

Therefore, our perturbation is $V = -zeE$.

In general, we can write our pertubed state as

$$|n\rangle = |n^{(0)}\rangle + \sum_{k \neq n} |k^{(0)}\rangle \frac{\langle k^{(0)}|V|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}} + \mathcal{O}(\lambda^2)$$

Neglecting our terms second order and higher, and using $n=0$ (the ground state):

$$|0\rangle = |0^{(0)}\rangle + \sum_{k \neq 0} |k^{(0)}\rangle \frac{\langle k^{(0)}|V|0^{(0)}\rangle}{E_0^{(0)} - E_k^{(0)}}$$

Assume this is a hydrogen atom, so the states are characterized by $|nlm\rangle$.
Appologizing for my bad notation, $|0\rangle = |n=1, l=0, m=0\rangle$ is the ground state.

$$|0\rangle = |100\rangle + \sum_{nlm \neq 100} |nlm\rangle \frac{\langle nlm|z|100\rangle}{E_{100} - E_{nlm}} (-eE)$$

Now, since $U = -\vec{p} \cdot \vec{E}$, and $\vec{p} = \vec{z}e$, $\vec{E} = E\hat{z}$ $\Rightarrow U = \langle 0|ze|0\rangle E$.

so $p = \langle 0|ze|0\rangle$

Let's evaluate this expectation value.

$$\langle 0|ze|0\rangle = \left[\langle 100| + \sum_{nlm} \langle nlm| \frac{\langle 100|z|nlm\rangle}{E_{100} - E_{nlm}}(-eE)\right] ez \left[|100\rangle + \sum_{nlm} |nlm\rangle \cdot (-eE) \frac{\langle nlm|z|100\rangle}{E_{100} - E_{nlm}}\right]$$

Neglect 2nd-order terms, and note $\langle 100|ez|100\rangle = 0$
b/c $l' \neq l+1$.

$$\boxed{p = \langle 0|ze|0\rangle = -2e^2 E \sum_{nlm} \frac{|\langle 100|z|nlm\rangle|^2}{E_{100} - E_{nlm}} = E\alpha}$$ by 5.1.68.

Now the energy shift computed to second order is given in the problem:

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$$\Delta = -\frac{1}{2}\alpha E^2 = E^2 e^2 \sum_{nlm} \frac{|\langle 100|z|nlm\rangle|^2}{E_{100} - E_{nlm}} = pE,$$ where once again $p = E\alpha$