$\boxed{5.41}$

Begining with 1.7.34b: $\phi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3\vec{x}\, e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{x}}\, \psi(\vec{x})$

In this case $\psi(x)$ is the groud state hydrogen atom wavefuction, given by Appendix B.6.7:

$$\langle r,\theta,\phi|100\rangle = R_{10}(r)\, Y_0^0(\theta,\phi) = R_{10}(r)\frac{1}{\sqrt{2\pi}} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$$

$$\Rightarrow \phi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \cdot \frac{1}{\sqrt{a_0^3\pi}} \int r^2 \sin\theta\, dr\, d\theta\, d\phi\; e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{r}}\, e^{-r/a_0}$$

$$= \frac{2\pi}{(2\pi\hbar)^{3/2}} \frac{1}{\sqrt{a_0^3\pi}} \int_0^{\pi}\int_0^{\infty} r^2 e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{r} - \frac{1}{a_0}r}\, dr\, \sin\theta\, d\theta.$$

Choose $\vec{p} = p\hat{z}$ (per problem statment) $\Rightarrow \vec{p}\cdot\vec{r} = pr\cos\theta$.

$$= \frac{1}{\pi\sqrt{2(a_0\hbar)^3}} \int_0^{\pi}\int_0^{\infty} r^2 e^{-\frac{i}{\hbar}pr\cos\theta - \frac{r}{a_0}} \sin\theta\, d\theta$$

$$= \frac{1}{\pi\sqrt{2(a_0\hbar)^3}} \frac{4a_0^3\hbar^4}{(a_0^2p^2+\hbar^2)^2} \quad \text{using mathematica.}$$

$$\Rightarrow \boxed{|\phi(\vec{p})|^2 d^3p' = \frac{8a_0^3\hbar^5}{\pi^2(a_0^2p^2+\hbar^2)^4} d^3p'}$$

$\frac{20}{20}$

$\frac{77}{100}$