Ben Levy
Quantum Mechanics II Problem set 2

Sakurai 4.8 and 4.10

$\boxed{4.8}$ (a) Given: $\hat{H}$ is invariant under time reversal ($\hat{\Theta}$), and part of a spinless, nondegenerate system.

Prove: The wavefunctions $\langle \vec{x} | n \rangle$ can be chosen to be $\in \mathbb{R}$.

Proof: If $\hat{H}$ is invariant under $\hat{\Theta}$, then

$$[\hat{H}, \hat{\Theta}] = 0 = \hat{H}\hat{\Theta} - \hat{\Theta}\hat{H} \Rightarrow \hat{H}\hat{\Theta} = \hat{\Theta}\hat{H}$$

Thus $\hat{H}\hat{\Theta}|n\rangle = \hat{\Theta}\hat{H}|n\rangle = E_n \hat{\Theta}|n\rangle$

So, the energy eigenvalue of $|n\rangle$ is $E_n$, and the energy eigenvalue of $\hat{\Theta}|n\rangle$ is <u>also</u> $E_n$.

Due to nondegeneracy, this means that $|n\rangle$ and $\hat{\Theta}|n\rangle$ must actually be the same state: $|n\rangle = \hat{\Theta}|n\rangle$.

In the position representation.

$$\langle \vec{x}|n\rangle = \langle \vec{x}|\hat{\Theta}|n\rangle = \langle \vec{x}|\hat{\Theta}\int d^3x\, |\vec{x}\rangle\langle \vec{x}|n\rangle$$
$$= \langle \vec{x}| \int d^3x\, |\vec{x}\rangle \langle \vec{x}|n\rangle^* \quad \text{by } 4.4.55$$
$$= \langle \vec{x}|n\rangle^*$$

We have the position-space wavefunction equal to it's complex conjugate.

(in general, if $r = x + iy$, then $r = r^* \Rightarrow x + iy = x - iy$
$\Rightarrow y = 0$)

Thus, $\langle \vec{x}|n\rangle$ is purely real. □ $\frac{20}{20}$

(b) Given: $\psi(t=0) = e^{i\vec{p}\cdot\vec{x}/\hbar}$

Why does this not violate time-reversal symmetry?

<u>Because</u> it must have degenerate energy eigenvalues. For example, there may be multiple states w/ differt $\vec{p}$, but the same $p^2$ (i.e. differt directions). Thus the Hamiltonian is the same for various states if $\hat{H} = \frac{p^2}{2m}$.

$\frac{20}{20}$