$\boxed{4.2}$ (a) $\mathcal{T}_d$ and $\mathcal{T}_{d'}$ obviously comute: 5 paces North + 2 paces East is the same as 2 paces East + 5 paces North. Let's show this.

$$\hat{\mathcal{T}}_d = 1 - \frac{i\vec{d}\cdot\hat{p}}{\hbar}, \quad \hat{\mathcal{T}}_{d'} = 1 - \frac{i\vec{d}'\cdot\hat{p}}{\hbar}$$

Now, let $\vec{d}\cdot\hat{p} \equiv P_i$ and $\vec{d}'\cdot\hat{p} \equiv P_j$

$$[\hat{\mathcal{T}}_d, \hat{\mathcal{T}}_{d'}] = \left(1 - \frac{iP_i}{\hbar}\right)\left(1 - \frac{iP_j}{\hbar}\right) - \left(1 - \frac{iP_j}{\hbar}\right)\left(1 - \frac{iP_i}{\hbar}\right)$$
$$= 1 - \frac{iP_i}{\hbar} - \frac{iP_j}{\hbar} - P_iP_j\frac{1}{\hbar^2} - 1 + \frac{i}{\hbar}P_j + \frac{i}{\hbar}P_i + \frac{1}{\hbar^2}P_jP_i$$
$$= \frac{1}{\hbar^2}(P_jP_i - P_iP_j) = 0 \quad \text{because } [P_j, P_i] = 0$$

$\boxed{\text{Yes. } \mathcal{T}_d \text{ and } \mathcal{T}_{d'} \text{ commute}}$

(b) Intuition tells me that $\hat{D}(\hat{n}, \phi)$ and $\hat{D}(\hat{n}', \phi')$ do not comute. Lets show this.

$$D(\hat{n}, \phi) \approx 1 - \frac{i\phi}{\hbar}\vec{J}\cdot\hat{n} = 1 - \frac{i\phi}{\hbar}J_i \quad (J_i \equiv \vec{J}\cdot\hat{n})$$
$$D(\hat{n}', \phi') \approx 1 - \frac{i}{\hbar}\phi'\vec{J}\cdot\hat{n}' = 1 - \frac{i\phi'}{\hbar}J_j \quad (J_j \equiv \vec{J}\cdot\hat{n}')$$
$$[\hat{D}(\hat{n}, \phi), \hat{D}(\hat{n}', \phi')] = 1 - \frac{i\phi}{\hbar}J_i - \frac{i}{\hbar}\phi' J_j - \frac{\phi\phi'}{\hbar^2}J_iJ_j - 1 + \frac{i\phi}{\hbar}J_i + \frac{i\phi'}{\hbar}J_j + \frac{\phi\phi'}{\hbar^2}J_jJ_i$$
$$= \frac{\phi\phi'}{\hbar^2}(J_jJ_i - J_iJ_j) = \frac{\phi\phi'}{\hbar^2}[J_j, J_i] \neq 0$$

b/c J_i and J_j do not comute: $\boxed{\text{No. } \hat{D}(\hat{n}, \phi), \hat{D}(\hat{n}', \phi') \text{ do not commute}}$

(c) $\mathcal{T}_d$ and $\hat{\pi}$ obviously do not commute: If I walk 5 paces forward, then turn around, I'm not at the same spot as turning around, and then walking 5 paces forward! Let's show this.

$$[\hat{\mathcal{T}}_d, \hat{\pi}] = \left(1 - \frac{i}{\hbar}\vec{d}\cdot\hat{p}\right)\pi - \pi\left(1 - \frac{i}{\hbar}\vec{d}\cdot\hat{p}\right)$$
$$= \pi - \frac{iP}{\hbar}\pi - \pi + \pi\frac{i}{\hbar}P = \frac{i}{\hbar}(\pi P - P\pi) \neq 0$$

b/c $\{\pi, P\} = 0 \Rightarrow [\pi, P] \neq 0$.

$\boxed{\text{No. } \mathcal{T}_d, \text{ and } \hat{\pi} \text{ do not commute.}}$

(d) $\hat{D}(\hat{n}, \phi)$ and $\hat{\pi}$ comute. Let's show this

$$[\hat{D}(\hat{n}, \phi), \hat{\pi}] = \left(1 - \frac{i\phi}{\hbar}J_n\right)\pi - \pi\left(1 - \frac{i\phi}{\hbar}J_n\right) = \pi - \frac{i\phi}{\hbar}J_n\pi - \pi + \pi\frac{i\phi}{\hbar}J_n$$
$$= \frac{i\phi}{\hbar}(\pi J_n - J_n\pi) = 0 \quad \text{b/c } [\pi, J_n] = 0 \quad \text{by 4.2.16.}$$

$\boxed{\text{Yes. } \hat{D}(\hat{n}, \phi) \text{ and } \hat{\pi} \text{ commute}}$

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