11.

Sakurai 3-9

We should begin by noting that

$$D^{(1/2)}(\alpha,\beta,\gamma) = \begin{bmatrix} e^{-i(\alpha+\gamma)/2}\cos(\frac{\beta}{2}) & -e^{-i(\alpha-\gamma)/2}\sin(\beta/2) \\ e^{i(\alpha-\gamma)/2}\sin(\beta/2) & e^{i(\alpha+\gamma)/2}\cos(\beta/2) \end{bmatrix}$$

has the form of the genral unimodular matrix in 3.3.7, with Cayley-Klein Parameters $a = e^{-i(\alpha+\gamma)/2}\cos(\frac{\beta}{2})$, $b = -e^{-i(\alpha-\gamma)/2}\sin(\beta/2)$.

→ Thus $D = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$, and D is unitary b/c $D^\dagger D = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix} = 1$.

→ D is also Unimodular because $aa^* - (b(-b^*)) = |a|^2 + |b|^2$

$$= \cancel{e^{-i(\alpha+\gamma)/2}}\cos(\tfrac{\beta}{2})\cancel{e^{i(\alpha+\gamma)/2}}\cos(\tfrac{\beta}{2}) + -\cancel{e^{-i(\alpha-\gamma)/2}}\sin(\beta/2)(-\cancel{e^{i(\alpha-\gamma)/2}})\sin(\beta/2)$$

$$= \cos^2(\tfrac{\beta}{2}) + \sin^2(\tfrac{\beta}{2}) = 1 \checkmark$$

Thus we use $\text{Re}(a) = \cos(\frac{\theta}{2}) \Rightarrow \theta = 2\cos^{-1}(\text{Re}(a))$ from 3.3.10

From Euler's Formula, $a = \left(\cos(\frac{\alpha+\gamma}{2}) - i\sin(\frac{\alpha+\gamma}{2})\right)\cos(\frac{\beta}{2})$ 10

So $\text{Re}(a) = \cos(\frac{\alpha+\gamma}{2})\cos(\frac{\beta}{2}) \Longrightarrow \checkmark\ \boxed{\theta = 2\cos^{-1}\left(\cos(\frac{\alpha+\gamma}{2})\cos(\frac{\beta}{2})\right)}$