Sakurai 3-8

First, we note by equation 3.1.1a (rewritten as a sumation), that $V_k' = \sum_l R_{kl} V_l$, where $k, l \in x, y, z$, is a rotation of vector $\vec{V}$ governed by 3x3 orthogonal rotation matrix R.

Now, given $U^{-1} A_k U = \sum_l R_{kl} A_l$, we realize that $U^{-1} A_k U = A_k'$ (e.g. 1.5.13). This is a similarity transformation — A_k' is a different matrix, but with all the same characteristics as A_k.

Thus $A_k' = \sum_l R_{kl} A_l$. As in 3.1.1a, this is just a rotation, this time of $\vec{A}$ — a vector of matricies $(\hat{A}_x, \hat{A}_y, \hat{A}_z)$ — governed by a rotation matrix R. The output is a new vector $\vec{A}'$ of matricies (A_k'). These would be A_x', A_y', A_z'.

Now we look at the individual elements of the matricies A_k: $\langle m | A_k | n \rangle$.

$$A_k' = \sum_l R_{kl} A_l \longrightarrow \langle m | A_k' | n \rangle = \sum_l R_{kl} \langle m | A_l | n \rangle \checkmark$$

If we compare to 3.1.1a, and see $V_k' \to \langle m | A_k' | n \rangle$, and $V_l \to \langle m | A_l | n \rangle$, then it is obvious that the matrix elements transform like vectors.

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