Sakurai 3-29

In the attached Mathematica document, I show that

$$\sum_{q'} d^{(1)}_{qq'}(\beta) V^{(1)}_{q'} = \begin{bmatrix} \frac{-1}{\sqrt{2}}(iV_y + V_x \cos(\beta) + V_z \sin(\beta)) \\ V_z \cos(\beta) - V_x \sin(\beta) \\ \frac{-1}{\sqrt{2}}(iV_y - V_x\cos(\beta) - V_z \sin(\beta)) \end{bmatrix}$$

This is exactly what we'd expect. Note that the rotation matrix about the y-axis by angle β is:

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \Rightarrow$$ under this rotation,

$V_x \to V_x \cos\beta + V_z \sin\beta$ ✓

$V_y \to V_y$ ✓

$V_z \to V_z \cos\beta - V_x \sin\beta$ ✓

10

Plugging those transformations into the definition of V:

$$V^{(1)}_+ = \frac{-(V_x + iV_y)}{\sqrt{2}} \rightarrow \frac{-(V_x\cos\beta + V_z \sin\beta + iV_y)}{\sqrt{2}}$$

$$V^{(1)}_0 = V_z \longrightarrow V_z\cos\beta - V_x \sin\beta$$

$$V^{(1)}_- = \frac{V_x - iV_y}{\sqrt{2}} \longrightarrow \frac{V_x\cos\beta + V_z\sin\beta - iV_y}{\sqrt{2}}$$

These are the exact same compounts of $\sum_{q'} d^{(1)}_{qq'}(\beta) V^{(1)}_{q'}$ that we found above. ✓

Define the $d^{(1)}_{q\,q'}$ matrix as d, and the V matrix $\mathbf{V} = \{V_+\ V_0\ V_-\}$, as V below.

In[65]:=

```
d = {{1/2 (1 + Cos[β]), -(1/√2) Sin[β], 1/2 (1 - Cos[β])},
   {(1/√2) Sin[β], Cos[β], -(1/√2) Sin[β]}, {1/2 (1 - Cos[β]), (1/√2) Sin[β], 1/2 (1 + Cos[β])}};
```

In[64]:=

```
V = {-((Vx + i Vy)/√2), Vz, ((Vx - i Vy)/√2)};
```

Now simply do the matrix multiplication as indicated to get the V' vector, rotated an angle β around the y axis.

In[69]:=

```
d.V // FullSimplify // MatrixForm
```

Out[69]//MatrixForm=

$$\begin{pmatrix} \frac{-i\,Vy - Vx\,Cos[\beta] - Vz\,Sin[\beta]}{\sqrt{2}} \\ Vz\,Cos[\beta] - Vx\,Sin[\beta] \\ \frac{-i\,Vy + Vx\,Cos[\beta] + Vz\,Sin[\beta]}{\sqrt{2}} \end{pmatrix}$$