Sakurai 3-2

(a) Before any measurements are made, particle one exists in a linear superposition of + and − states. What observer A measures in each case will be completely random.

In other words we could either write

$$|\text{spin singlet}\rangle = \left(\tfrac{1}{\sqrt{2}}\right)\left(|\hat{z}+, \hat{z}-\rangle - |\hat{z}-, \hat{z}+\rangle\right)$$

$$\text{or } |\text{spin singlet}\rangle = \left(\tfrac{1}{\sqrt{2}}\right)\left(|\hat{x}-, \hat{x}+\rangle - |\hat{x}+, \hat{x}-\rangle\right)$$

Thus

$$P(S_{1z}|\text{spin singlet}\rangle = +\tfrac{\hbar}{2}|\text{singlet state}\rangle) = \tfrac{1}{2}$$

$$P(S_{1x}|\text{spin singlet}\rangle = +\tfrac{\hbar}{2}|\text{singlet state}\rangle) = \tfrac{1}{2}$$ ✓

There is a 50% chance A measures $\hbar/2$ in either case

(b) Now obs. B measures that particle 2 is z-spin up.

(i) in order to maintain $S_{total} = 0$, $\boxed{S_{1z} = -\hbar/2}$ ✓

(That is, in the first equation above, we are in the $|\hat{z}-, \hat{z}+\rangle$ state w/ certainty as soon as B measures $S_{z2} = +\hbar/2$)

(ii) What will observer A get if she measures $S_{1x}$?

using 3.2.52, w/ $\beta = \pi/2$ (b/c there is a 90° angle between the $+\hat{z}$ and $+\hat{x}$ directions), and $\alpha = 0$ (b/c we are in a plane), we get:

$$|\text{particle } 1\rangle = \cos\left(\tfrac{\beta}{2}\right)e^{-i\alpha/2}|\hat{x}+\rangle + \sin\left(\tfrac{\beta}{2}\right)e^{-i\alpha/2}|\hat{x}-\rangle$$

$$= \cos(\pi/4)|\hat{x}+\rangle + \sin(\tfrac{\pi}{4})|\hat{x}-\rangle$$

$$= \tfrac{1}{\sqrt{2}}\left(|\hat{x}+\rangle + |\hat{x}-\rangle\right)$$ ✓ 10

Thus, there are equal probabilities for measuring $S_{1x} = +\hbar/2$ and $S_{1x} = -\hbar/2$.