sabusai 3-20 We are looking for the follow 3 probabilities: $|D^{(2)}_{0,0}(\alpha,\beta,\gamma)|^2$, $|D^{(2)}_{\pm1\,0}(\alpha,\beta,\gamma)|^2$, and $|D^{(2)}_{\pm2}(\alpha,\beta,\gamma)|^2$, where $\alpha=\gamma=0$, and $\beta\neq 0$. These are the probabilities that after $|l=2, m=0\rangle$ is rotated by β it will be found in $m=0, \pm1, \pm2$ respectively.

First, lets look up the following Spherical Harmonics.

$$Y_2^0(\beta,0) = \sqrt{\tfrac{5}{16\pi}}(3\cos^2\beta-1), \quad Y_2^{\pm1}(\beta,0) = \mp\sqrt{\tfrac{15}{8\pi}}(\sin\beta\cos\beta)e^{\pm i0}$$

$$Y_2^{\pm2}(\beta,0) = \mp\sqrt{\tfrac{15}{8\pi}}(\sin\beta\cos\beta)e^{\pm i0}$$

Then, by 3.6.51, $|D^{(2)}_{0,0}(0,\beta,0)|^2 = \left|\sqrt{\tfrac{4\pi}{5}}Y_2^0(\beta,0)\right|^2 = \boxed{\overset{m=0}{\tfrac{1}{4}(3\cos^2\beta-1)^2}}$

for $m=\pm1$ $|D^{(2)}_{\pm1,0}(0,\beta,0)|^2 = \left|\sqrt{\tfrac{4\pi}{5}}Y_2^{\pm1}(\beta,0)\right|^2 = \boxed{\overset{m=\pm1}{\tfrac{3}{2}\sin^2\beta\cos^2\beta}}$

for $m=\pm2$ $|D^{(2)}_{\pm2,0}(0,\beta,0)|^2 = \left|\sqrt{\tfrac{4\pi}{5}}Y_2^{\pm2}(\beta,0)\right|^2 = \boxed{\overset{m=\pm2}{\tfrac{3}{8}\sin^4(\beta)}}$

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