Sakurai 2-4

$P(\nu_e \to \nu_e) = |\langle \nu_e | U(t_0=0, t) | \nu_e \rangle|^2$

given $|\nu_e\rangle = \cos\theta\,|\nu_1\rangle - \sin\theta\,|\nu_2\rangle$

and $U(t_0=0,t) = \exp[-iHt/\hbar] \approx \exp\left[\frac{-it}{\hbar}\left(pc + \frac{m^2pc^3}{2p^2}\right)\right]$

with $m=m_1$ for $U(t)|\nu_1\rangle$ and $m=m_2$ for $U(t)|\nu_2\rangle$.

On the attached peice of scratch paper, I used (See Next Page)

$\Delta m^2 = m_1^2 - m_2^2$, $E=pc$, and $L=ct$ to calculate

$$\boxed{P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\sin^2\left(\Delta m^2 c^4 \frac{L}{4E\hbar c}\right)}$$ ✓

The amplitude of the curve is about 0.35, and the amplitude of the equation is $\sin^2(2\theta) = 0.35 \Rightarrow \theta = 0.32$ radians or $\boxed{\approx 18^\circ}$ ✓

The Period of the curve is about 35 L/E, and the frequency of the equation is $\frac{\Delta m^2 c^4}{4\hbar c}$. $T = \frac{2\pi}{f}$

Thus $\Delta m^2 c^4 = \frac{8\pi\hbar c}{35\, L/E} = \frac{8\pi \cdot 197\,\text{eV nm}}{35\,\frac{\text{km}}{\text{MeV}}}$

$= \frac{8\pi \cdot 197\,\text{eV}\,\cancel{\text{nm}}}{3.5\cdot 10^7\,\frac{\cancel{\text{nm}}}{\text{eV}}} = \boxed{0.00014\,\text{eV}^2}$ ✓

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$$|\nu_e(t)\rangle = \cos\theta \exp\left[\frac{-it}{\hbar}\left(pc + \frac{m_1^2 pc^3}{2p^2}\right)\right]|\nu_1\rangle - \sin\theta \exp\left[\frac{-it}{\hbar}\left(pc + \frac{m_2^2 pc^3}{2p^2}\right)\right]|\nu_2\rangle$$

$$\langle \nu_e | \nu_e(t)\rangle = \cos^2\theta \exp\Bigg[\underbrace{\frac{-it}{\hbar}\left(pc + \frac{m_1^2 pc^3}{2p^2}\right)}_{x_1}\Bigg] + \sin^2\theta \exp\Bigg[\underbrace{\frac{-it}{\hbar}\left(pc + \frac{m_2^2 pc^3}{2p^2}\right)}_{x_2}\Bigg]$$

$$|\langle \nu_e|\nu_e(t)\rangle|^2 = \cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta \exp[-i(x_1) + ix_2] + \cos^2\theta\sin^2\theta \exp[-ix_2 + ix_1]$$

$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta +$$

$$= (\cos^2\theta + \sin^2\theta)^2 + \cos^2\theta\sin^2\theta\left[-2 + \exp[-i(x_1 - x_2)] + \exp[i(x_1 - x_2)]\right]$$

$$= 1 + \frac{\sin^2(2\theta)}{4}\left[-2 + 2\cos(x_1 - x_2)\right]$$

$$= 1 - \sin^2(2\theta)\left[\frac{1}{2} + \frac{\cos(x_1 - x_2)}{2}\right]$$

$$= 1 - \sin^2(2\theta)\left[\sin^2\left(\frac{x_1 - x_2}{2}\right)\right]$$

$$= 1 - \sin^2(2\theta)\sin^2\left(\frac{\frac{t}{\hbar}\frac{m_1^2 pc^3}{2p^2} - \frac{t}{\hbar}\frac{m_2^2 pc^3}{2p^2}}{2}\right)$$

$$= 1 - \sin^2(2\theta)\sin^2\left(\frac{t}{\hbar}\frac{\Delta m^2 pc^3}{4p^2}\right)$$

$$= 1 - \sin^2(2\theta)\sin^2\left(\Delta m^2 c^4 \frac{ptc}{\hbar 4 p^2 c^2}\right)$$

$$= 1 - \sin^2(2\theta)\sin^2\left(\Delta m^2 c^4 \frac{pL}{4pcE\hbar}\right)$$

$$|\langle \nu_e|\nu_e(t)\rangle|^2 = \boxed{1 - \sin^2(2\theta)\sin^2\left(\Delta m^2 c^4 \frac{L}{4E\hbar c}\right)}$$