Given: $\vec{B} = \nabla \times A$

$\vec{B} = B_0 \hat{z}$ $\quad \vec{P} \to \vec{P} - \frac{e\vec{A}}{c}$

QM 6.3

2.38

Sakurai 2-38

W

2.7.26

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + e\phi = \frac{1}{2m}\left(\vec{P}\cdot\vec{P} - \frac{e}{c}\vec{P}\cdot\vec{A} - \frac{e}{c}\vec{A}\cdot\vec{P} + \frac{e^2}{c^2}\vec{A}\cdot\vec{A}\right) + e\phi$$

$$= \frac{1}{2m}\left(\vec{P}\cdot\vec{P} - \frac{e}{c}\left(P_x A_x + P_y A_y + P_z A_z + A_x P_x + A_y P_y + A_z P_z\right) + \left(\frac{e}{c}\right)^2\left(A_x^2 + A_y^2 + A_z^2\right)\right) + e\phi$$

$(\vec{\nabla} \times \vec{A})_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} A_k$, we know $B_x = B_y = 0 \Rightarrow (\vec{\nabla} \times \vec{A})_x = 0 = \epsilon_{xyz} \frac{\partial}{\partial y} A_z = \frac{\partial}{\partial y} A_z$

$B_0 \hat{z} = \vec{\nabla} \times \vec{A}$ $\quad$ and $(\vec{\nabla} \times \vec{A})_y = 0 = \epsilon_{yxz} \frac{\partial}{\partial x} A_z = -\frac{\partial}{\partial x} A_z$

$A_z = 0$ since $\vec{\nabla} \times \vec{A} = \vec{B}$ $\quad$ also $A_x = -\frac{B_0 y}{2}$, $A_y = \frac{B_0 x}{2}$ (2.7.40) ✓

$$H = \frac{1}{2m}\left(\vec{P}\cdot\vec{P} - \frac{e}{c}\left(P_x A_x + P_y A_y + A_x P_x + A_y P_y\right) + \left(\frac{e}{c}\right)^2\left(A_x^2 + A_y^2\right)\right) + e\phi.$$

Note $\left\{ (A_x^2 + A_y^2) = \frac{B_0^2 y^2}{4} + \frac{B_0^2 x^2}{4} = \frac{B_0^2}{4}(x^2 + y^2) \right.$

So Plug that in $\rightarrow$

$$H = \frac{1}{2m}\left(\vec{P}\cdot\vec{P} - \frac{e}{c}\left(-P_x \frac{B_0 y}{2} + P_y \frac{B_0 x}{2} - \frac{B_0 y}{2} P_x + \frac{B_0 x}{2} P_y\right) + \left(\frac{e B_0}{2c}\right)^2 (x^2 + y^2)\right) + e\phi$$

$$\frac{2e}{c}\left(\frac{B_0}{2}\left(x P_y - y P_x\right)\right) = \frac{2e}{c}\left(\frac{B_0}{2} L_z\right) = \frac{e B_0 L_z}{c}$$

we knew that $(x P_y - y P_x) = L_z$ by taking the cross product: $\vec{L} = \vec{x} \times \vec{P} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix} = \underbrace{(y P_z - z P_y)}_{L_x}\hat{x} - \underbrace{(x P_z - z P_x)}_{L_y}\hat{y} + \underbrace{(x P_y - y P_x)}_{L_z}\hat{z}$

Now plug this term back into the hamiltonian:

$$H = \frac{\vec{P}\cdot\vec{P}}{2m} - \frac{e B_0}{2mc} L_z + \frac{1}{2m}\left(\frac{e B_0}{2c}\right)^2 (x^2 + y^2) + e\phi$$

$$\boxed{H = \frac{\vec{P}^2}{2m} - \frac{e B_0}{2mc} L_z + \frac{e^2}{8mc^2} B_0^2 (x^2 + y^2) + e\phi} \checkmark$$

interaction of $\vec{B}$ w/ $\mu = \mu_z = \frac{-e}{2mc} L_z$ $\qquad$ prop. to $B_0^2 (x^2 + y^2)$

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