$\boxed{\text{Sakurai 2-30}}$ We begin with the expression for probability flux (2.4.16)

$\rightarrow \vec{j}(\vec{x},t) = -\left(\frac{i\hbar}{2m}\right)[\psi^* \vec{\nabla}\psi - (\vec{\nabla}\psi^*)\psi] = \left(\frac{\hbar}{m}\right)\text{Im}(\psi^* \vec{\nabla}\psi)$

$\rightarrow$ From Appendix B.6 we see $\psi_{n\ell m} = R_{n\ell}(r) Y_\ell^m(\theta,\phi)$ for hydrogen atoms.

Furthermore, $Y_\ell^m(\theta,\phi) = \left(\sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\right) P_\ell^{|m|}\cos(\theta) e^{im\phi}$

Define the constant as $A_{\ell m}$. Then $\psi_{n\ell m} = R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}\cos(\theta)\, e^{im\phi}$

$\rightarrow$ In spherical coordinates $\vec{\nabla} = \nabla_r \hat{r} + \nabla_\theta \hat{\theta} + \nabla_\phi \hat{\phi} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}$

Let's consider $\vec{j}$ by components:

$j_r = \frac{\hbar}{m_e}(\text{Im}(\psi^* \nabla_r \psi)) = \frac{\hbar}{m}\text{Im}\left(R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}\cos(\theta) \cancel{e^{-im\phi}} \frac{\partial}{\partial r} R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}\cos(\theta) \cancel{e^{+im\phi}} \right)$

$= \frac{\hbar}{m_e}\text{Im}\left(R_{n\ell}(r) \frac{\partial}{\partial r}(R_{n\ell}(r)) \left(A_{\ell m} P_\ell^{|m|}\cos(\theta)\right)^2 \right) = 0$

Because R, A, and P are all real. ✓

$j_\theta = \frac{\hbar}{m_e}\text{Im}(\psi^* \nabla_\theta \psi) = \frac{\hbar}{m}\text{Im}\left(R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}(\cos(\theta)) \cancel{e^{-im\phi}} R_{n\ell}(r) A_{\ell m} \frac{1}{r}\frac{\partial}{\partial\theta} P_\ell^{|m|}(\cos\theta) \cancel{e^{+im\phi}} \right)$

$= \frac{\hbar}{m_e}\text{Im}\left((R_{n\ell}(r) A_{\ell m})^2 \frac{1}{r} P_\ell^{|m|}(\cos(\theta)) \frac{\partial}{\partial r} P_\ell^{|m|}(\cos\theta) \right) = 0$

again b/c R, A, and P are all real. ✓

Since $j_\theta = j_r = 0$, $\vec{j} = j_\phi \hat{\phi}$

$j_\phi = \frac{\hbar}{m_e}\text{Im}(\psi^* \nabla_\phi \psi) = \frac{\hbar}{m}\text{Im}\left[R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}(\cos\theta) e^{-im\phi} \frac{R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}(\cos\theta)}{r\sin\theta} \underbrace{ime^{im\phi}}_{\frac{\partial}{\partial\phi}e^{im\phi}} \right]$

$\vec{j} = \frac{\hbar m}{m_e r\sin\theta}\left(R_{n\ell}(r) A_{\ell m} P_\ell^{|m|}(\cos\theta)\right)^2 = \boxed{\frac{\hbar m_\ell |\psi_{n\ell m}|^2}{m_e r\sin\theta}\hat{\phi}}$ ✓

Thus, when $m_\ell \neq 0$, the flux flows in the $\pm\hat{\phi}$ direction.

when $m_\ell > 0$, $\vec{j}$ is in the $+\hat{\phi}$ direction

when $m_\ell < 0$, $\vec{j}$ is in the $-\hat{\phi}$ direction.

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