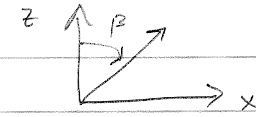

sakurai 2-3

(a) Given $H = -\left(\frac{e}{m_e c}\right) S \cdot B\hat{z} = -\left(\frac{e}{m_e c}\right) S_z = \omega S_z$

Now notice that our initial state is exactly the situation in homework problem 1-9 with $\alpha \to 0$

Thus @ $t=0$ $\quad |S_n +\rangle = \cos\left(\frac{\beta}{2}\right)|+\rangle + \sin\left(\frac{\beta}{2}\right)|-\rangle$

Now time evolve:

$|S_n^+, t\rangle = \mathcal{U}(t, t_0=0)|S_n^+\rangle = \exp\left[\frac{-iHt}{\hbar}\right]|S_n^+\rangle = \exp\left[\frac{-i\omega S_z t}{\hbar}\right]|S_n^+\rangle$

Since $|S_n^+\rangle$ isn't an eigenstate of the S_z operator, we need to write it in terms of $|+\rangle$ and $|-\rangle$. Luckily I just wrote how to do this above:

$= \exp\left[\frac{-i\omega S_z t}{\hbar}\right]\cos\left(\frac{\beta}{2}\right)|+\rangle + \exp\left[\frac{-i\omega S_z t}{\hbar}\right]\sin\left(\frac{\beta}{2}\right)|-\rangle$

$= \exp\left[\frac{-i\omega t}{2}\right]\cos\left(\frac{\beta}{2}\right)|+\rangle + \exp\left[\frac{i\omega t}{2}\right]\sin\left(\frac{\beta}{2}\right)|-\rangle = |S_n^+, t\rangle$

The probability of finding $S_x = \frac{\hbar}{2}$ $(|S_x+\rangle)$ @ time t, we look for

$|\langle S_x^+ | S_n^+, t\rangle|^2 = \left|\left[\frac{1}{\sqrt{2}}\langle +| + \frac{1}{\sqrt{2}}\langle -|\right]\left[\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/\hbar}|+\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\omega t/\hbar}|-\rangle\right]\right|^2$

$= \left|\frac{1}{\sqrt{2}}\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/\hbar} + \frac{1}{\sqrt{2}}\sin\left(\frac{\beta}{2}\right)e^{i\omega t/\hbar}\right|^2$

$= \left|\frac{1}{\sqrt{2}}\cos\left(\frac{\beta}{2}\right)\left(\cos\left(\frac{\omega t}{2}\right) - i\sin\left(\frac{\omega t}{2}\right)\right) + \frac{1}{\sqrt{2}}\left(\sin\left(\frac{\beta}{2}\right)\left(\cos\left(\frac{\omega t}{2}\right) + i\sin\left(\frac{\omega t}{2}\right)\right)\right)\right|^2$

$= \frac{1}{2}\left(\cos\frac{\beta}{2}\sin\frac{\beta}{2}e^{i\omega t} + \cos\frac{\beta}{2}\sin\frac{\beta}{2}e^{-i\omega t}\right) = \boxed{\frac{1}{2}\left(1 + \sin(\beta)\cos(\omega t)\right)}$ ✓

(b) The easy way to do this is to multiply the probabilities of all possible states by the eigenvalue, and sum.

$\langle S_x\rangle_t = \left(\frac{\hbar}{2}\right)\frac{1}{2}\left(1+\sin(\beta)\cos(\omega t)\right) + \left(\frac{-\hbar}{2}\right)\left(1 - \frac{1}{2}\left(1+\sin\beta\cos(\omega t)\right)\right)$ ← $1 - |\langle S_x^+|S_n^+, t\rangle|^2$

$= \frac{\hbar}{2}\left[\frac{1}{2} + \frac{1}{2}\sin\beta\cos(\omega t) - \frac{1}{2} + \frac{1}{2}\sin(\beta)\cos(\omega t)\right]$

$= \boxed{\frac{\hbar}{2}\left(\sin(\beta)\cos(\omega t)\right)}$ ✓

(c) (i) if $\beta \to 0$

→ Then $\boxed{|\langle S_x^+|S_n^+, t\rangle|^2 = \frac{1}{2}}$ which makes sense b/c $|S_n^+\rangle \to |S_z^+\rangle$, and we expect to find the e^- not precessing since it's already alligned. (Just like original Stern-Gelarch expeint)

✓

→ then $\boxed{\langle S_x\rangle_t = 0.}$ This makes sense: have of the time $|S_x^+\rangle$, half $|S_x^-\rangle$

(ii) if $\beta \to \frac{\pi}{2}$

→ Then $\boxed{|\langle S_x^+|S_n^+, t\rangle|^2 = \frac{1}{2}\left(1+\cos(\omega t)\right)}$ @ $t=0$, $t = \frac{2\pi}{\omega}$, etc

This equals 1 because $|S_x^+\rangle = |S_n^+, t\rangle$ as one would want.

→ Then $\boxed{\langle S_x\rangle_t = \frac{\hbar}{2}\cos(\omega t)}$ Simply in x-y plane, precessing around z-axis.

My mind is @ peace...

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