Sabwai 2-14a

(a) Given: $\left.\begin{matrix} a \\ a^+ \end{matrix}\right\} = \sqrt{\frac{m\omega}{2\hbar}}\left(x \pm \frac{ip}{m\omega}\right), \quad \left.\begin{matrix} a|n\rangle \\ a^+|n\rangle \end{matrix}\right\} = \begin{cases} \sqrt{n}\,|n-1\rangle \\ \sqrt{n+1}\,|n+1\rangle \end{cases}$

Now we must calculate the following five quantities:

$$\langle m|x|n\rangle \overset{2.3.24}{=} \sqrt{\frac{\hbar}{2m\omega}}\langle m|(a+a^+)|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\langle m|a|n\rangle + \langle m|a^+|n\rangle\right)$$
$$= \sqrt{\frac{\hbar}{2m\omega}}\left(\langle m|\sqrt{n}|n-1\rangle + \langle m|\sqrt{n+1}|n+1\rangle\right) = \boxed{\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\,\delta_{m,n-1} + \sqrt{n+1}\,\delta_{m,n+1}\right)}$$

$$\langle m|p|n\rangle \overset{2.3.24}{=} i\sqrt{\frac{m\hbar\omega}{2}}\left(\langle m|(-a+a^+)|n\rangle\right) = i\sqrt{\frac{m\hbar\omega}{2}}\left(-\langle m|\sqrt{n}|n-1\rangle + \langle m|\sqrt{n+1}|n+1\rangle\right)$$
$$= \boxed{i\sqrt{\frac{m\hbar\omega}{2}}\left(-\sqrt{n}\,\delta_{m,n-1} + \sqrt{n+1}\,\delta_{m,n+1}\right)}$$

$$\langle m|\{x,p\}|n\rangle = \langle m|xp|n\rangle + \langle m|px|n\rangle = \frac{i\hbar}{2}\langle m|(a+a^+)(-a+a^+)|n\rangle + \frac{i\hbar}{2}\langle m|(-a+a^+)(a+a^+)|n\rangle$$
$$= \frac{i\hbar}{2}\left[\langle m|-aa|n\rangle + \cancel{\langle m|(-a^+a)|n\rangle} + \cancel{\langle m|aa^+|n\rangle} + \langle m|a^+a^+|n\rangle\right.$$
$$\left. + \langle m|(-aa)|n\rangle + \cancel{\langle m|a^+a|n\rangle} + \cancel{\langle m|(-aa^+)|n\rangle} + \langle m|a^+a^+|n\rangle\right]$$
$$= i\hbar\left[\langle m|(-aa)|n\rangle + \langle m|a^+a^+)|n\rangle\right] = i\hbar\left[-\langle m|a\sqrt{n}|n-1\rangle + \langle m|a^+\sqrt{n+1}|n+1\rangle\right]$$
$$= i\hbar\left[-\sqrt{n}\langle m|\sqrt{n-1}|n-2\rangle + \sqrt{n+1}\langle m|\sqrt{n+2}|n+2\rangle\right]$$
$$= \boxed{-i\hbar\left(\sqrt{n(n-1)}\,\delta_{m,n-2} - \sqrt{(n+1)(n+2)}\,\delta_{m,n+2}\right)}$$

$$\langle m|xx|n\rangle = \left(\sqrt{\frac{\hbar}{2m\omega}}\right)^2\langle m|(a+a^+)(a+a^+)|n\rangle$$
$$= \left[\langle m|aa|n\rangle + \langle m|a^+a^+|n\rangle + \langle m|a^+a|n\rangle + \langle m|aa^+|n\rangle\right]\frac{\hbar}{2m\omega}$$
$$= \frac{\hbar}{2m\omega}\left[\sqrt{n(n-1)}\,\delta_{m,n-2} + \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n}\sqrt{n}\langle m|n\rangle + \sqrt{(n+1)^2}\langle m|n\rangle\right]$$
$$= \boxed{\frac{\hbar}{2m\omega}\left[\sqrt{n(n-1)}\,\delta_{m,n-2} + \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + (2n+1)\,\delta_{m,n}\right]}$$

$$\langle m|pp|n\rangle = \left(i\sqrt{\frac{m\hbar\omega}{2}}\right)^2\langle m|(-a+a^+)(-a+a^+)|n\rangle$$
$$= \left(i\sqrt{\frac{m\hbar\omega}{2}}\right)^2\left[\langle m|aa|n\rangle + \langle m|a^+a^+|n\rangle + \langle m|(-aa^+)|n\rangle + \langle m|(-a^+a)|n\rangle\right]$$
$$= -\left(\frac{\hbar m\omega}{2}\right)\left[\sqrt{n(n-1)}\,\delta_{m,n-2} + \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} - (n+1)\delta_{m,n} - n\,\delta_{m,n}\right]$$
$$= \boxed{\frac{\hbar m\omega}{2}\left[-\sqrt{n(n-1)}\,\delta_{m,n-2} - \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + (2n+1)\,\delta_{m,n}\right]}$$

10