Sakurai 1.5

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(a) We are to find the matrix representation of $|\alpha\rangle\langle\beta|$ in the $\{\langle a'|\}$ basis. given $\langle a'|\alpha\rangle, \langle a''|\alpha\rangle, \ldots$ and $\langle a'|\beta\rangle, \langle a''|\beta\rangle, \ldots$

We are given $|\alpha\rangle$ & $|\beta\rangle$ w/ a set of basis kets.

The matrix elems then can be written $\langle a'|\alpha\rangle\langle\beta|a''\rangle$

and the matrix is

$$\boxed{|\alpha\rangle\langle\beta| = \begin{bmatrix} \langle a^{(1)}|\alpha\rangle\langle\beta|a^{(1)}\rangle & \cdots & \langle a^{(1)}|\alpha\rangle\langle\beta|a^{(n)}\rangle \\ \vdots & \ddots & \vdots \\ \langle a^{(n)}|\alpha\rangle\langle\beta|a^{(1)}\rangle & \cdots & \langle a^{(n)}|\alpha\rangle\langle\beta|a^{(n)}\rangle \end{bmatrix}}$$ ✓

(b) now $|\alpha\rangle \rightarrow |S_z^+\rangle \rightarrow |+\rangle$

and $|\beta\rangle \rightarrow |S_x^+\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \Rightarrow \langle S_x^+| = \frac{1}{\sqrt{2}}\langle+| + \frac{1}{\sqrt{2}}\langle-|$

Now our matrix is

$$\begin{bmatrix} \langle+|\alpha\rangle\langle\beta|+\rangle & \langle+|\alpha\rangle\langle\beta|-\rangle \\ \langle-|\alpha\rangle\langle\beta|+\rangle & \langle-|\alpha\rangle\langle\beta|-\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\langle+|+\rangle}_{1}\langle S_x^+|+\rangle & \underbrace{\langle+|+\rangle}_{1}\langle S_x^+|-\rangle \\ \underbrace{\langle-|+\rangle}_{0}\langle S_x^+|+\rangle & \underbrace{\langle-|+\rangle}_{0}\langle S_x^+|-\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{bmatrix}$$

✓

$$\Rightarrow |S_z^+\rangle\langle S_x^+| = \boxed{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}$$