Sakurai 1-33

$\boxed{ai}$ $\langle p'|x|\alpha\rangle = \langle p'|x \int dx\, |x'\rangle\langle x'|\alpha\rangle = \int dx\, \langle p'|x|x'\rangle\langle x'|\alpha\rangle$

$= \int x'\, dx\, \langle p'|x'\rangle\langle x'|\alpha\rangle \overset{1.7.32}{=} \frac{1}{\sqrt{2\pi\hbar}} \int dx'\, x'\, e^{-ip'x'/\hbar} \langle x'|\alpha\rangle$

Now Note $i\hbar \frac{\partial}{\partial p'} e^{-ip'x'/\hbar} = i\hbar \left(\frac{-ix'}{\hbar}\right) e^{-ip'x'/\hbar} = x' e^{-ip'x'/\hbar}$

So, we can plug this into integral:

$\langle p'|x|\alpha\rangle = \int dx'\, i\hbar \frac{\partial}{\partial p'} \langle x'|\alpha\rangle e^{-ip'x'/\hbar}$

$= i\hbar \frac{\partial}{\partial p'} \frac{1}{\sqrt{2\pi\hbar}} \int dx'\, \langle x'|\alpha\rangle e^{-ip'x'/\hbar}$ ✓

$= i\hbar \frac{\partial}{\partial p'} \frac{1}{\sqrt{2\pi\hbar}} \int dx'\, e^{-ip'x'/\hbar} \psi_\alpha(x') \overset{1.7.34b}{=} \boxed{i\hbar \frac{\partial}{\partial p'} \langle p'|\alpha\rangle}$

$\boxed{aii}$ $\langle \beta|x|\alpha\rangle = \langle\beta| \int dp' |p'\rangle\langle p'|x|\alpha\rangle = \langle\beta| \int dp'\, |p'\rangle i\hbar \frac{\partial}{\partial p'} \langle p'|\alpha\rangle$ ✓

$= \int dp'\, \langle\beta|p'\rangle i\hbar \frac{\partial}{\partial p'} \langle p'|\alpha\rangle = \boxed{\int dp'\, \phi_\beta^*(p')\, i\hbar\, \phi_\alpha(p)}$

$\boxed{b}$ $\exp\left[\frac{ix\Xi}{\hbar}\right]$ looks a lot like a translation in position space, but w/ $+i$ and an x operator instead of a p operator. Maybe it's a translation in momentum space. Let's see: ✓

$\exp\left[\frac{ix\Xi}{\hbar}\right]|p\rangle = \int dx \exp\left[\frac{ix\Xi}{\hbar}\right] |x\rangle\langle x|p\rangle$ completeness

plug in 1.7.32 for $\langle x|p\rangle$

$= \int dx \exp\left[\frac{ix\Xi}{\hbar}\right] \exp\left[\frac{ipx'}{\hbar}\right] \frac{1}{\sqrt{2\pi\hbar}} |x\rangle = \int dx \exp\left[\frac{i(p+\Xi)x}{\hbar}\right] |x\rangle$

$= \int dx\, |x\rangle \underbrace{\frac{1}{\sqrt{2\pi\hbar}} e^{i(p+\Xi)x/\hbar}}$

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This is simply $\langle x|p+\Xi\rangle$ from 1.7.32 again.

undo completeness: $\boxed{\exp\left[\frac{ix\Xi}{\hbar}\right]|p\rangle = |p+\Xi\rangle}$. Yes. Generator of translations in momentum space. ✓