Sakurai 1-29

ⓐ $[x_i, G(\vec{p})] = [x_i, G(p_i)]$ because we can always consolidate the other terms (p_j, p_k, etc...) will always commute w/ x_i, and those terms will go to zero. → ~~They'll stick around, but they will not be~~ affected by differentiation with respect to x_i

Taylor series:

$$G(p_i) = \sum_{n=0}^{\infty} g_n p_i^n$$

Note:

$$[A, B^n] = n B^{n-1} [A, B] \rightarrow$$ -1 where did this come from? (can be proved for x_i and p_i by induction)

$$[x_i, G(\vec{p})] = [x_i, \sum_{n=0}^{\infty} g_n p_i^n] = [x_i, g_1 p_i] + [x_i, g_2 p_i^2] + \sum_{n=3}^{\infty}[x_i, g_n p_i^n]$$

$$= \sum_{n=0}^{\infty} g_n [x_i, p_i^n] = \sum_{n=0}^{\infty} g_n \underbrace{n p_i^{(n-1)} [x_i, p_i]}$$

← see above identity.

$$= i\hbar \sum_{n=0}^{\infty} n g_n p_i^{(n-1)} = i\hbar \frac{\partial G}{\partial p_i} \quad \square$$

$$\left(\text{because } \frac{\partial}{\partial p_i} G = \frac{\partial}{\partial p_i} \sum_{n=0}^{\infty} g_n p_i^n = \sum_{n=0}^{\infty} g_n n p_i^{n-1}\right)$$

Likewise

$$[p_i, F(\vec{x})] = [p_i, F(x_i)] = [p_i, \sum_{n=0}^{\infty} f_n x_i^n] = \sum_{n=0}^{\infty} f_n [p_i, x_i^n] = \sum_{n=0}^{\infty} f_n n x_i^{n-1} [p_i, x_i]$$

$$= -i\hbar \underbrace{\sum_{n=0}^{\infty} f_n n x_i^{n-1}} = -i\hbar \frac{\partial F}{\partial x_i} \quad \square$$

as before

ⓑ $$[x^2, p^2] = [x^2, p]p + p[x^2, p] = x[x,p]p + [x,p]xp + px[x,p] + p[x,p]x$$

$$= x i\hbar p + i\hbar x p + p x i\hbar + p i\hbar x$$

$$= i\hbar(2xp + 2px) = \boxed{2i\hbar(xp + px)}$$

$$[x^2, p^2]_{classical} = \underbrace{\frac{\partial x^2}{\partial x}\frac{\partial p^2}{\partial p} - \frac{\cancel{\partial x^2}}{\partial p}\frac{\cancel{\partial p^2}}{\partial x}}_{} = 2x \cdot 2p = 4xp$$

0 0

This is by 1.6.48 which states:

$$[A(q,p), B(q,p)]_{classical} = \sum_s \left(\frac{\partial A}{\partial q_s}\frac{\partial B}{\partial p_s} - \frac{\partial A}{\partial p_s}\frac{\partial B}{\partial q_s}\right)$$

(Poisson brackets! You'll encounter those in classical mechanics soon.)

where $A \to x^2$, $B \to p^2$, $q \to x$, $p \to p$ (duh), and we have just one s.

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