Solution 13

Here is a drawing of the described series of SG apparatuses. I leave off the initial SG-z apparatus as we normalize it's output (i.e. 100% of particles start out in the $|S_z^+\rangle$ state.

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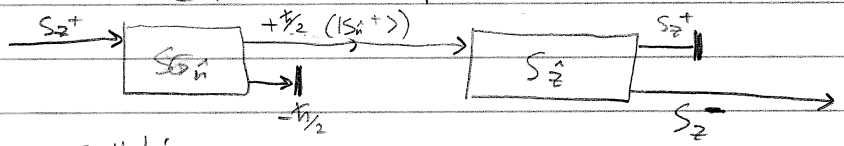

Good news #1:

Intensity (I) that goes through successive apparatus is the same as the probabolities of a particle getting through in successive experiments. Since the experiments are successive we multiply the probability of an $|S_n^+\rangle$ outcome given the initial $|S_z^+\rangle$ state (i.e. $|\langle S_z^+|S_n^+\rangle|^2$), and the probability of a $|S_z^-\rangle$ outcome given that $|S_n^+\rangle$ intermediate outcome; ($|\langle S_n^+|S_z^-\rangle|^2$): $\quad I = |\langle S_n^+|S_z^-\rangle|^2 |\langle S_z^+|S_n^+\rangle|^2$

Good news #2: in problem 9, we worked out $|S_n^+\rangle = \cos\left(\frac{\beta}{2}\right)|+\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}|-\rangle$ in the S_z- basis.

$$I = \langle S_n^+|S_z^-\rangle\langle S_z^-|S_n^+\rangle\langle S_z^+|S_n^+\rangle\langle S_n^+|S_z^+\rangle$$

$$= \left[\cos\tfrac{\beta}{2}\langle+|-\rangle + \sin\tfrac{\beta}{2}e^{-i\alpha}\langle-|-\rangle\right]\cdot\left[\cos\tfrac{\beta}{2}\langle-|+\rangle + \sin\left(\tfrac{\beta}{2}\right)e^{i\alpha}\langle-|-\rangle\right]$$
$$\cdot\left[\cos\tfrac{\beta}{2}\langle+|+\rangle + \sin\tfrac{\beta}{2}e^{i\alpha}\langle+|-\rangle\right]\cdot\left[\cos\tfrac{\beta}{2}\langle+|+\rangle + \sin\tfrac{\beta}{2}e^{-i\alpha}\langle-|+\rangle\right]$$

$$= \left(\sin\tfrac{\beta}{2}e^{-i\alpha}\right)\left(\sin\tfrac{\beta}{2}e^{i\alpha}\right)\left(\cos\tfrac{\beta}{2}\right)\left(\cos\tfrac{\beta}{2}\right) \quad ✓$$

$$= \sin^2\left(\tfrac{\beta}{2}\right)\cos^2\left(\tfrac{\beta}{2}\right) = \frac{1-\cos\beta}{2}\cdot\frac{1+\cos\beta}{2} = \frac{(1-\cos\beta)^2}{4} \rightarrow \boxed{I = \frac{\sin^2\beta}{4}}$$

We should orient the $SG_{\hat{n}}$ apparatus at $\beta = \frac{\pi}{2}$ radians relative to the z-axis. This maximizes the intensity to $\boxed{0.25}$ of the beam that entered the $SG_{\hat{n}}$ apparatus. ✓