7.14

We need density of states

$$\Sigma(p) = \frac{1}{h^3}\int d^3q\, d^3p = \frac{V}{h^3}\frac{4\pi}{3}p^3 \longrightarrow \frac{1}{h^n}\int d^nq\, d^np = \frac{V p^n}{h^n}\frac{\pi^{n/2}}{(n/2)!}$$

$$g(p)dp = \frac{d\Sigma(p)}{dp}dp = \frac{nVp^{n-1}}{h^n}\frac{\pi^{n/2}}{(n/2)!}dp$$

$\varepsilon \propto p^s \Rightarrow \varepsilon = Ap^s$, where A is a constant $\Rightarrow p^n = \left(\frac{\varepsilon}{A}\right)^{n/s}$

$$dp = d\varepsilon \cdot \frac{dp}{d\varepsilon} = \frac{n\left(\frac{\varepsilon}{A}\right)^{-1+n/s}}{As}d\varepsilon = \frac{n\left(\frac{\varepsilon}{A}\right)^{n/s}}{\varepsilon s}d\varepsilon$$

(★) $$a(\varepsilon)d\varepsilon = \frac{V\pi^{n/2}}{h^n (n/2)!} \times \frac{n\left(\frac{\varepsilon}{A}\right)^{n/s}}{\varepsilon s}d\varepsilon = \frac{V}{h^n}\frac{n}{s}\frac{\varepsilon^{n/s-1}}{A^{n/s}}\frac{\pi^{n/2}}{(n/2)!}d\varepsilon$$

Replace 7.1.4 with (★), and proceed with 7.1.5

$$\frac{P}{kT} = \frac{V}{h^s}\frac{n}{s}\frac{\pi^{n/2}}{A^{n/s}(n/2)!}\int_0^\infty \varepsilon^{n/s-1}\underbrace{\ln(1-ze^{-\beta\varepsilon})}_{u}d\varepsilon$$ (taking $T\to\infty$, we neglect the inconsequential $\varepsilon=0$ term)

$dv \Rightarrow v = \frac{s}{n}\varepsilon^{n/s}$

$$= \frac{V}{h^s}\frac{n}{sA^{n/s}}\frac{\pi^{n/2}}{(n/2)!}\left(\ln(1-ze^{-\beta\varepsilon})\frac{s}{n}\varepsilon^{n/s}\Big|_0^\infty - \frac{s}{n}\int_0^\infty \varepsilon^{n/s}\frac{z\beta e^{-\beta\varepsilon}}{1-ze^{-\beta\varepsilon}}d\varepsilon\right)$$

$$= \frac{\beta z V}{h^s A^{n/s}}\frac{n}{s}\frac{\pi^{n/2}}{(n/2)!}\int_0^\infty \varepsilon^{n/s}\frac{e^{-\beta\varepsilon}}{1-ze^{-\beta\varepsilon}}d\varepsilon = \frac{V\beta}{h^s A^{n/s}}\left(\frac{n}{s}\frac{\pi^{n/2}}{(n/2)!}\right)\int_0^\infty \frac{\varepsilon^{n/s}}{z^{-1}e^{\beta\varepsilon}-1}d\varepsilon$$

use Bose functions to take the integral. Let $x = \beta\varepsilon \Rightarrow dx = \beta d\varepsilon$

$$= \frac{V\cancel{\beta}}{\beta^{n/s}h^s A^{n/s}}\left(\frac{n}{s}\frac{\pi^{n/2}}{(n/2)!}\right)\int_0^\infty \frac{x^{n/s}}{z^{-1}e^{x}-1}\frac{dx}{\cancel{\beta}} = \frac{V}{h^s(A\beta)^{n/s}}\left(\frac{n}{s}\frac{\pi^{n/2}}{(n/2)!}\right)g_{\frac{n}{s}+1}(z)\,\Gamma\!\left(\frac{n}{s}+1\right)$$

Thermodynamics

by 7.1.11 $U = kT^2\frac{\partial}{\partial T}\left(\frac{PV}{kT}\right)\Big|_{z,V}$ note $\frac{PV}{kT} \sim \frac{1}{\beta^{n/s}} = (kT)^{n/s} \Rightarrow \frac{\partial}{\partial T}\left(\frac{PV}{kT}\right) \sim \frac{n}{s}(kT)^{n/s-1}$

$$\Rightarrow \frac{\partial}{\partial T}\left(\frac{PV}{kT}\right) = \frac{n}{Ts}\left(\frac{PV}{kT}\right) \Rightarrow U = \cancel{kT^2}\frac{n}{s}\frac{PV}{\cancel{kT^2}} \Rightarrow \frac{U}{V} = \frac{n}{s}P \Rightarrow \boxed{P = \frac{s}{n}\frac{U}{V}}$$

In the $T\to\infty$ limit, I think this looks like an ideal gas, where $PV = NkT$.

$$P = \frac{s}{n}\frac{U}{V} \Rightarrow U = \frac{n}{s}VP = \frac{nNkT}{s}$$

By 7.1.15, $C_V = \left(\frac{\partial U}{\partial T}\right)_{N,V} = \boxed{\frac{n}{s}Nk = C_V}$

Also for ideal gasses,

$C_P = C_V + Nk$, so $\boxed{C_P = \left(\frac{n}{s}+1\right)Nk}$

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