6.8 Assume our cylinder to have height $L \to \infty$, and cross-sectional area A.

The hamiltonian of each of the N particles contains a kinetic & potential term:

$$H = \frac{p^2}{2m} + mgz$$

$$\Rightarrow Q_1 = \sum_{\varepsilon} e^{-\beta\varepsilon} \xrightarrow{\text{transform to 3D phase space}} Q_1 = \frac{1}{h^3}\int e^{-\beta\left(\frac{p^2}{2m}+mgz\right)} d^3p\, d^3q$$

$$= \frac{A}{h^3}\int_{-\infty}^{\infty} e^{-\frac{\beta p^2}{2m}} d^3p \int_0^L e^{-\beta mgz} dz = A\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\left(\frac{1-e^{-\beta mgL}}{mg\beta}\right)$$

In the limit where $L \to \infty$, $Q = \frac{A}{mg\beta h^3}(2\pi mkT)^{3/2}$

$$\boxed{Q_N = \frac{1}{N!}(Q_1)^N = \frac{1}{N!}\left(\frac{AkT}{mgh^3}(2\pi mkT)^{3/2}\right)^N}$$

The helmholtz free energy (which I will call F here) can be written as follows:

$$F = -kT\ln Q_N = -kT\left(N\ln\left[\frac{A}{mg\beta h^3}(2\pi mkT)^{3/2}\right] - N\ln[N]\right)$$

$$= -NkT\ln\left[\frac{A}{N}\frac{1}{mg\beta h^3}(2\pi mkT)^{3/2}\right]$$

I'll write F in terms of β and kT:

$$F(\beta, N) = -\frac{N}{\beta}\ln\left[\frac{A}{N}\frac{\beta^{-5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right]$$

$$F(T, N) = -NkT\ln\left[\frac{A}{N}\frac{(kT)^{5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right]$$

Major Thermodynamic properties:

$$U = -\frac{\partial}{\partial\beta}\Big((-\beta)F(\beta,N)\Big) = \frac{-\partial}{\partial\beta}\left(\frac{N}{\beta}\ln\left[\frac{A}{N}\frac{\beta^{-5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right]\right) = \frac{5N}{2\beta} \Rightarrow \boxed{U = \frac{5}{2}NkT}$$

$$S = -\frac{\partial}{\partial T}\big(F(T,N)\big) = \frac{\partial}{\partial T}\left(NkT\ln\left[\frac{A}{N}\frac{(kT)^{5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right]\right)$$

$$= \frac{5kN}{2} + kN\ln\left(\frac{A}{N}\frac{(kT)^{5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right) \Rightarrow \boxed{S = \frac{5}{2}Nk - \frac{F}{T}}$$

$$\mu = \frac{\partial}{\partial N}\big(F(T,N)\big) = kT - kT\ln\left[\frac{A}{N}\frac{(kT)^{5/2}}{mg}\left(\frac{2\pi m}{h^2}\right)^{3/2}\right] \Rightarrow \boxed{\mu = kT + \frac{F}{N}}$$

By sending $L \to \infty$, we erased all info about the overall pressure (in z-direction). We can still calculate a "2D pressure" (Force per unit length) parallel to the x-y plane:

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$$P = -\frac{\partial}{\partial A}F(T,N,A) = \boxed{\frac{kTN}{A} = P} \leftarrow \text{"lateral pressure"}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{N,V} = \boxed{\frac{5}{2}Nk}$$

Increasing temperature requires the average particle height to increase, thus requiring additional potential energy to be added beyond the free-space case.

#4 Pathria Problem 6.8

The Helmholtz Free Energy (F) is given by the following in terms of β:

```
F[β_, N_] = -N/β (Log[A/N (β)^(-5/2)/(m g) (2 π m/h^2)^(3/2)]);
```

$$F[\beta\_,\ N\_] = -\frac{N}{\beta}\left(\mathrm{Log}\left[\frac{A}{N}\,\frac{(\beta)^{-5/2}}{m\,g}\left(\frac{2\,\pi\,m}{h^2}\right)^{3/2}\right]\right);$$

Use Helmholtz Free Energy to calculate the Internal Energy

```
-D[-β*F[β, N], β]
```

$$\frac{5\,N}{2\,\beta}$$

In terms of kT, The Helmholtz Free Energy (F) is given by:

$$F[T\_,\ N\_,\ A\_] = -N\,k\,T\left(\mathrm{Log}\left[\frac{A}{N}\,\frac{(k\,T)^{5/2}}{m\,g}\left(\frac{2\,\pi\,m}{h^2}\right)^{3/2}\right]\right);$$

Calculate Entropy:

```
-D[F[T, N, A], T]
```

$$\frac{5\,k\,N}{2} + k\,N\,\mathrm{Log}\left[\frac{2\,\sqrt{2}\,A\left(\frac{m}{h^2}\right)^{3/2}\pi^{3/2}\,(k\,T)^{5/2}}{g\,m\,N}\right]$$

Calculate μ:

```
D[F[T, N, A], N]
```

$$k\,T - k\,T\,\mathrm{Log}\left[\frac{2\,\sqrt{2}\,A\left(\frac{m}{h^2}\right)^{3/2}\pi^{3/2}\,(k\,T)^{5/2}}{g\,m\,N}\right]$$