3.31 A fermi Oscillator has the very simple energy eigenvalues $\varepsilon_n = n\cdot\varepsilon \; (n=0,1)$

$$Q(\beta) = \sum_{n=0}^{1} e^{-\beta n \varepsilon} = 1+e^{-\beta\varepsilon} \Rightarrow Q_N(\beta) = [Q_1(\beta)]^N = (1+e^{-\beta\varepsilon})^N$$

Now that we have the partion function, I'll calculate the thermodynamics of the system of N oscillators:

$\rightarrow A = -kT\ln(Q_N(\beta)) = -kTN\ln(1+e^{-\beta\varepsilon}) \Rightarrow \boxed{A = -kTN\ln(1+e^{-\varepsilon/kT})}$

$\rightarrow \mu = \frac{A}{N} \Rightarrow \boxed{\mu = -kT\ln(1+e^{-\beta\varepsilon})}$

$\rightarrow$ also, $\boxed{P=0}$ b/c the oscillators can't explore the volume they occupy.

$\rightarrow S = -\frac{\partial A}{\partial T}\Big|_N = kN\ln(1+e^{-\beta\varepsilon}) + \frac{kTN}{1+e^{-\varepsilon/kT}} \cdot \frac{\varepsilon e^{-\varepsilon/kT}}{kT^2}$

$\Rightarrow \boxed{S = Nk\ln(1+e^{-\beta\varepsilon}) + \frac{N\varepsilon e^{-\beta\varepsilon}}{T(1+e^{-\beta\varepsilon})}}$

$\rightarrow U = A+TS = \cancel{-kTN\ln(1+e^{-\varepsilon/kT})} + \cancel{TNk\ln(1+e^{-\beta\varepsilon})} + \frac{N\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

$\Rightarrow \boxed{U = \frac{N\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}}$

Usually, for oscillators, the # of energy quata availible for distribution among N oscillators is $R = \frac{(E-\frac{1}{2}N\hbar\omega)}{\hbar\omega}$ where E is the total energy. In this case the zero-point energy $\frac{1}{2}\hbar\omega \rightarrow 0$, and the energy quata $\hbar\omega \rightarrow \varepsilon$. $\therefore$ $R = \frac{E}{\varepsilon}$

The multiplicty of states, then, reduces to the # of ways to distribute R quata into N boxes, where we allow no more than 1 quata per box.

$$\boxed{\Omega = \frac{N!}{R!(N-R)!}}$$

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We can use Ω to calculate the thermodynamics as well.

$S = k\ln\Omega = k\ln N! - k\ln R! - k\ln(N-R)!$

where $N \gg R \gg 1$, we use stirling's Approx: $S \approx kN\ln N - kR\ln R - k(N-R)\ln(N-R)$

$S = kN\ln\left[\frac{N}{N-R}\right] + kR\ln\left[\frac{N-R}{R}\right] \Rightarrow \boxed{S \approx kN\ln\left[\frac{N\varepsilon}{N\varepsilon - E}\right] + \frac{kE}{\varepsilon}\ln\left[\frac{N\varepsilon - E}{E}\right]}$

$T = 1\Big/\frac{\partial S}{\partial E}\Big|_N \Rightarrow \boxed{T = \frac{\varepsilon}{k\ln\left(\frac{\varepsilon k}{E} - 1\right)}}$ mathematica for derivative $\Rightarrow \beta\varepsilon = \ln\left(\frac{\varepsilon N}{E} - 1\right) \Rightarrow e^{\beta\varepsilon} = \frac{\varepsilon N}{E} - 1$

$\therefore \boxed{\frac{E}{N} = \frac{\varepsilon}{1+e^{\beta\varepsilon}}}$ $\rightarrow$ classical limit is where $T \gg 1$.

Then $\frac{E}{N} \approx \varepsilon$ (most of the oscillators are excited, as expected

$\rightarrow$ In the $T \ll 1$ limit, $E \rightarrow 0$ also as expected