$\boxed{3.26}$ $\varepsilon_j = (j + \frac{s}{2})\hbar\omega = (j + \sum_{i=1}^{s} \frac{1}{2})\hbar\omega = j\hbar\omega + \sum_{i=1}^{s} \frac{1}{2}\hbar\omega$

Now, the $j\hbar\omega$ portion of the energy must be split up among the s dimensions. In other words $j\hbar\omega = \sum_{i=1}^{s} l_i \hbar\omega$, where $l_i\hbar\omega$ is the contribution from dimension i to the j^{th} energy level (beyond the zero-point contribution $\frac{1}{2}\hbar\omega$).

$$\Rightarrow \varepsilon_j = \sum_{i=1}^{s} (l_i + \tfrac{1}{2})\hbar\omega$$

This is simply the same situation as s 1-D simple harmonic oscillators, which is an example worked by the book. We have j distinct quata which must be divied up amongst s "oscillators" 3.8.25 gives us the multiplicity: $\boxed{\frac{(j+s-1)!}{j!(s-1)!} = g(E)}$.

Partition Function

Then $Q_1(\beta) = \sum_{j=0}^{\infty} g(E) e^{-\beta\varepsilon_j} = \sum_{j=0}^{\infty} g(E) e^{-\beta(j+\frac{s}{2})\hbar\omega} = e^{-\beta\frac{s}{2}\hbar\omega} \sum_{j=0}^{\infty} \frac{(j+s-1)!}{j!(s-1)!} e^{-\beta j\hbar\omega}$

Simplify using Mathematica (see attached)

$$Q_1(\beta) = \frac{e^{-\beta\frac{s}{2}\hbar\omega}}{(1-e^{-\beta\hbar\omega})^s} \Rightarrow Q_N = (Q_1(\beta))^N = \boxed{\frac{e^{-\beta N\frac{s}{2}\hbar\omega}}{(1-e^{-\beta\hbar\omega})^{sN}}}$$

Thermodynamic Properties (s-D oscillators, so use "s" subscript)

$$A = -kT\ln(Q_N) = -kT[-\beta N\tfrac{s}{2}\hbar\omega - sN\ln(1-e^{-\beta\hbar\omega})]$$

$$= sN\tfrac{1}{2}\hbar\omega + kTsN\ln(1-e^{-\beta\hbar\omega}) \qquad (\text{since } \beta = \tfrac{1}{kT})$$

$$\therefore \boxed{A_s = sN[\tfrac{1}{2}\hbar\omega + kT\ln(1-e^{-\beta\hbar\omega})]}$$

$$\mu = \frac{\partial S}{\partial N} = \boxed{s[\tfrac{1}{2}\hbar\omega + kT\ln(1-e^{-\beta\hbar\omega})] = \mu_s} \qquad \boxed{P_s = 0}$$

← Isolated, non-mobile oscillators.

$$S = \frac{-\partial A}{\partial T} = \boxed{sNk\left[\frac{\beta\hbar\omega}{e^{\beta\hbar\omega}-1} - \ln\left(1-e^{-\beta\hbar\omega}\right)\right] = S_s}$$

← Differentiation using Mathematica.

Compare with sN 1-D oscillators (use subscript "1")

(These are all worked out on p. 67 — just set $N \to sN$)

$$A_1 = sN[\tfrac{1}{2}\hbar\omega + kT\ln(1-e^{-\beta\hbar\omega})] = A_s \Rightarrow \boxed{A_1 = A_s}$$

$$\mu_1 = \frac{A_1}{sN} = [\tfrac{1}{2}\hbar\omega + kT\ln(1-e^{-\beta\hbar\omega})] = \frac{\mu_s}{s} \Rightarrow \boxed{\mu_s = s\mu_1}$$

$\boxed{P_1 = P_s}$

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$$S_1 = \frac{-\partial A_1}{\partial T} = \frac{-\partial A_s}{\partial T} = S_s \Rightarrow \boxed{S_1 = S_s}$$

Just a simplification of the partition function:

$$\sum_{j=0}^{\infty} \frac{(j+s-1)!}{j! * (s-1)!} \mathrm{Exp}[-\beta\, j\, \hbar\, \omega]$$

$$\left(1 - e^{-\beta\, \omega\, \hbar}\right)^{-s}$$

Use Helmholtz Free Energy to calculate the Entropy

$$\mathrm{A[T\_]} = s\, N \left(\frac{1}{2} \hbar\, \omega + k\, T\, \mathrm{Log}\left[1 - e^{-\frac{\hbar\, \omega}{k\, T}}\right]\right);$$

$$-\mathrm{D[A[T], T]}\ //\ \mathrm{FullSimplify}$$

$$s\, N \left(\frac{\omega\, \hbar}{\left(-1 + e^{\frac{\omega\, \hbar}{k\, T}}\right) T} - k\, \mathrm{Log}\left[1 - e^{-\frac{\omega\, \hbar}{k\, T}}\right]\right)$$