$\boxed{2\text{-}8}$ We are given that $2 = \int_0^\infty e^{-r_i} r_i^2 \, dr_i$

"Following my nose," it seems advantageous to multiply N copies of this identity together:

$$2^N = \prod_{i=1}^{N} \int_0^\infty e^{-r_i} r_i^2 \, dr_i = \int_0^\infty e^{\sum_{i=1}^{N} r_i} \left(\prod_{i=1}^{N} r_i^2 \, dr_i \right)$$

Now multiply by $1 = \frac{(4\pi)^N}{(4\pi)^N}$

$$2^N = \frac{1}{(4\pi)^N} \int_0^\infty e^{-\sum_{i=1}^{N} r_i} \left(\prod_{i=1}^{N} 4\pi r_i^2 \, dr_i \right)$$

Next, define $\sum_{i=1}^{N} r_i = R$, and note that the surface area of a hypersphere $S(R) = \prod_{i=1}^{N} (4\pi r_i^2)$

Meanwhile $\prod_{i=1}^{N} dr_i = d^n r = dR$

$$2^N = \frac{1}{(4\pi)^N} \int_0^\infty e^{-R} S(R) \, dR$$ and by C.3, $S_n(R) = n C_n R^{n-1}$

Since we are looking for V_{3N}, $n \Rightarrow 3N$, and $S_{3N}(R) = 3N C_{3N} R^{3N-1}$

$$2^N = \frac{3N C_{3N}}{(4\pi)^N} \int_0^\infty e^{-R} R^{3N-1} \, dR$$

To evaluate the integral note that $\int_0^\infty e^{-x} x^k \, dx = k!$

$$\therefore \; 2^N = \frac{3N C_{3N}}{(4\pi)^N} (3N-1)! \Rightarrow C_{3N} = \frac{(8\pi)^N}{3N(3N-1)!} = \frac{(8\pi)^N}{(3N)!}$$

Finally C.2 proclaims $V_n(R) = C_n R^n$

Thus $\boxed{V_{3N} = \frac{(8\pi)^N R^{3N}}{(3N)!}}$

In order to find thermodynamic quantities such as entropy, we need multiplicity: Γ

by 2.3.5, $\Gamma = \frac{\omega}{\omega_0}$, and for this 3N-dimensional system, $\omega_0 = h^{3N}$

$$\Gamma = \frac{(8\pi)^N (R/h)^{3N}}{(3N)!} V^N$$ ← from the integral over position

By 2.3.6, $S = k \ln \Gamma = k \ln \left[\frac{(8\pi (R/h)^3)^N}{(3N)!} V^N \right]$

Implicitly, $S = S(N, V, E)$. We would like to make the energy-dependance explicit so we can invert the function (i.e. solve for E).

$\varepsilon_i = p_i c \Rightarrow E = \sum_{i=1}^{3N} \varepsilon_i = c \sum_{i=1}^{3N} p_i \Rightarrow \sum_{i=1}^{3N} p_i = \frac{E}{c} \equiv R.$ ← clearly, this a purely-momentum phase space!

2-8 cont.

$$S = k \ln \left[\frac{\left(V 8\pi \left(\frac{E}{ch}\right)^3\right)^N}{(3N)!} \right] \Rightarrow e^{S/k} = \frac{(8\pi)^N}{(3N)!} \left(\frac{E}{ch}\right)^{3N} V^N$$

$$\Rightarrow \left[V^N (ch)^{3N} \frac{(3N)!}{(8\pi)^N} e^{S/k} \right]^{1/3N} = E(S, V, N)$$

$$P = -\left(\frac{\partial E}{\partial V}\right)_{N,S} = \frac{1}{3V} \left[\left(\frac{ch}{V}\right)^{3N} \frac{(3N)!}{(8\pi)^N} e^{S/k} \right]^{1/3N} \quad \boxed{P = \frac{E}{3V}} \qquad \boxed{PV = \frac{E}{3}}$$

$$PV = NkT \Rightarrow T = \frac{PV}{3Nk} = \frac{E}{3Nk} \Rightarrow \boxed{T = \frac{E}{3Nk}}$$

Thus $E = 3NkT$ so $C_V = \left(\frac{\partial E}{\partial T}\right)_{N,V} = \boxed{3Nk = C_V}$

and $C_p = \left(\frac{\partial (E + \overbrace{PV}^{E/3})}{\partial T}\right)_{N,P} = \frac{\partial \left(3NkT + \frac{3NkT}{3}\right)}{\partial T} = \boxed{4Nk = C_p}$

So $\frac{C_p}{C_V} = \frac{4Nk}{3Nk} \Rightarrow \boxed{C_p / C_V = \frac{4}{3}}$ Exactly as in 1.7

Also, $\mu = \left(\frac{\partial E}{\partial N}\right)_{V,S} = \boxed{3kT = \mu}$

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