Pathria 1-3

We are given the following system with a fixed partition That is able to transmit energy and particles:

A	B
$V_A = $ const.	$V_B = $ const.
S_A E_A N_A	S_B E_B N_B
μ_A T_A	μ_B T_B

The total energy and # of particles are conserved: $E \equiv E_a + E_B = \text{const.}$

$N \equiv N_A + N_B = \text{const.}$

Further, entropy adds: $S \equiv S_A + S_B$

Show: $\left(\frac{dE_A}{dN_A}\right)_{min} = \frac{\mu_A T_B - \mu_B T_A}{T_B - T_A}$

Begin w/ (1.3.4): $dE = TdS - PdV + \mu dN$

we can also make versions for the two halves:

$$dE_A = T_A dS_A - P_A dV_A + \mu_A dN_A$$

$$dE_B = T_B dS_B - P_B dV_B + \mu_B dN_B$$

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In each case $dV = dV_A = dV_B = 0$ (container is fixed).

(1) $dE = TdS + \mu dN$

(2) $dE_A = T_A dS_A + \mu_A dN_A$

(3) $dE_B = T_B dS_B + \mu_B dN_B$

using $E = E_A + E_B$, $\Rightarrow$ $\underset{0}{\cancel{dE}} - dE_A = \mu_B \underset{0}{\cancel{dN}} - \mu_B dN_A + T_B dS - T_B dS_A$ (E=const., N=const.).

(4) $-dE_A = -\mu_B dN_A + T_B dS - T_B dS_A$

Solve both (2) and (4) for dS_A, and equate them:

$$\frac{dE_A - \mu_A dN_A}{T_A} = \frac{-dE_A - T_B dS + \mu_B dN_A}{-T_B}$$

Simplify: $-dE_A T_B + \mu_B T_B dN_A = -dE_A T_A - T_B T_A dS + \mu_B T_A dN_A$

$dE_A(T_A - T_B) + dN_A(\mu_A T - \mu_B T_A) = -T_A T_B dS$

$$dE_A = \frac{(\mu_A T_B - \mu_B T_A)}{T_B - T_A} dN_A + \frac{T_A T_B}{T_B - T_A} dS$$

If $T_B > T_A$, then $\frac{T_A T_B}{T_B - T_A} dS \geq 0$ b/c $dS \geq 0$ by the 2^{nd} law.

Thus if T_A is lower

$$dE_A \leq \frac{\mu_A T_B - \mu_B T_A}{T_B - T_A} dN_A \Rightarrow \boxed{\left(\frac{dE_A}{dN_A}\right)_{min} = \frac{\mu_A T_B - \mu_B T_A}{T_B - T_A}} \quad \square$$